The structure for unifying multiple modeling alternatives (SUMMA), Version 1.0: Technical Description

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NCAR Technical Note.
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Abstract

This note describes the conservation equations and flux parameterizations used in the Structure for Unifying Multiple Modeling Alternatives (SUMMA). The processes considered here include radiation transfer through the vegetation canopy, within- and below-canopy turbulence, canopy interception, canopy transpiration, snow accumulation and ablation, and runoff generation.
1. Introduction

This work builds on the emerging literature of modeling methodologies that integrate multiple modeling alternatives [Moore and Clarke, 1981; Beven and Freer, 2001; Leavesley et al., 2002; Pomeroy et al., 2007; Clark et al., 2008; Fenicia et al., 2011; Niu et al., 2011; Essery et al., 2013] by introducing a new modeling framework, the Structure for Unifying Multiple Modeling Alternatives (SUMMA), for systematic analysis of competing modeling options. In developing SUMMA, we identify a common set of conservation equations (shared across multiple existing models), and make a set of general spatial approximations that preserve the flexibility to experiment with different modeling alternatives [Clark et al., 2015a; Clark et al., 2015b].

SUMMA advances existing modeling frameworks by (i) supporting different model representations of spatial variability and hydrologic connectivity, enabling analysis of the effects of the spatial discretization approach and the representation of lateral flow processes on basin-wide runoff and evapotranspiration fluxes; (ii) supporting a broad range of biophysical and hydrologic modeling options, enabling analysis of both the impacts of model simplification and the impacts of the choice of modeling approaches for individual physical processes; (iii) supporting analysis of a broad range of model parameter values, providing flexibility to evaluate the interplay between the choice of model parameters and the choice of process parameterizations; and (iv) separating modeling decisions on process representation from their numerical implementation, providing capabilities to experiment with different numerical solvers.

In this note we specify the conservation equations in SUMMA along with the multiple parameterizations for the different thermodynamic and hydrologic fluxes. The presentation provides complete details of the flux parameterization used in the first version of SUMMA, and as such this Technical Note can be used as an addendum to the model development presented in Clark et al. [2015a; 2015b]. The presentation is kept general to accommodate multiple physical processes representations within a single modeling framework.

2. Conservation equations

In presenting the model equations we use the following rules to maintain a consistent notation. All vertical fluxes are defined as positive downward. Turbulent energy fluxes from the canopy to the canopy air space are defined as positive towards the canopy, and lateral fluxes of liquid water within the soil profile and subterranean aquifer are defined as positive downslope. Superscripts are used to define the model sub-domain (the superscripts \textit{cas}, \textit{veg}, \textit{snow} and \textit{soil} denote the canopy air space, the vegetation canopy, snow, and soil respectively, and the superscript \textit{ss} denotes the snow and soil sub-domain). Subscripts are used to define additional characteristics as needed, such as the type of constituent
within a model sub-domain (the subscripts $\text{liq}$, $\text{ice}$, $\text{veg}$, $\text{soil}$ and $\text{air}$ denote the constituents of liquid water, ice, vegetation, soil, and air). The variable $z$ (m) defines the vertical coordinate (positive downward) and $h$ (m) defines the height above the soil surface (positive upwards), where $z = 0$ and $h = 0$ define the height of the soil surface. The abbreviations LHS and RHS are used to refer to the left-hand-side and right-hand-side of the model equations.

2.1 Thermodynamics

The thermodynamic state of the system depends on radiative energy fluxes through the vegetation canopy, turbulent heat fluxes within the canopy, and diffusion of heat throughout the snow-soil system. We only consider energy fluxes in the vertical dimension (lateral energy fluxes are assumed to be zero).

The conservation equations for thermodynamics are given as [Clark et al., 2015b]

\[
C_p^\text{cas} \frac{\partial T^\text{cas}}{\partial t} = -\frac{\partial Q_h^\text{cas}}{\partial z} - H^\text{veg} \tag{1}
\]

\[
C_p^\text{veg} \frac{\partial T^\text{veg}}{\partial t} - \rho_{\text{ice}} L_{\text{fus}} \left[ \frac{\partial \theta_{\text{ice}}^\text{veg}}{\partial t} \right]_{mf} = -\frac{\partial \left( Q_{\text{sud}} + Q_{\text{lwu}} + Q_{\text{swd}} + Q_{\text{lwu}} + Q_{\text{p}} \right)}{\partial z} + H^\text{veg} + H^\text{veg} \tag{2}
\]

\[
C_s^\text{ss} \frac{\partial T^\text{ss}}{\partial t} - \rho_{\text{ice}} L_{\text{fus}} \left[ \frac{\partial \theta_{\text{ice}}^\text{ss}}{\partial t} \right]_{mf} = -\frac{\partial F}{\partial z} \tag{3}
\]

where equations (1), (2) and (3) define the conservation equations for the temperature of the canopy air space ($\text{cas}$), the temperature of the vegetation canopy ($\text{veg}$) and the temperature of the snow and soil domain ($\text{ss}$). The first and second terms on the LHS of equations (2) and (3) define, respectively, the rate of change of temperature $T$ (K) and the rate of change of volumetric ice content $\theta_{\text{ice}}$ (-) associated with melt-freeze processes (as defined by the subscript $mf$), where an increase in $\theta_{\text{ice}}$ is freezing and a decrease is melting. In equations (1) through (3), $C_p$ (J m$^{-3}$ K$^{-1}$) is the volumetric heat capacity, and $L_{\text{fus}}$ (J kg$^{-1}$) and $\rho_{\text{ice}}$ (kg m$^{-3}$) are physical constants defining the latent heat of fusion and the intrinsic density of ice.

The RHS of equations (1) through (3) define the energy fluxes, where $Q_h^\text{cas}$ (W m$^{-2}$) is the vertical flux of sensible heat within the canopy air space, $Q_{\text{sud}}$ and $Q_{\text{lwu}}$ (W m$^{-2}$) are the downwelling shortwave and longwave radiation fluxes, $Q_{\text{swd}}$ and $Q_{\text{lwu}}$ (W m$^{-2}$) are the upwelling shortwave and longwave radiation
fluxes, $Q_p$ (W m$^{-2}$) is heat advected with precipitation, $H_{\text{sen}}^{\text{veg}}$ (W m$^{-3}$) and $H_{\text{lat}}^{\text{veg}}$ (W m$^{-3}$) are the volumetric sensible and latent heat fluxes, respectively, from the vegetation elements to the air space within the canopy ($H_{\text{sen}}^{\text{veg}}$ and $H_{\text{lat}}^{\text{veg}}$ are defined as positive towards the vegetation elements), and $F$ (W m$^{-2}$) is the vertical energy flux within the snow and soil domains.

The boundary conditions for equation (1) can be given as

$$Q_{h}^{\text{cas}} = \begin{cases} Q_{h}^{\text{total}}, & z = -h_{\text{top}} \\ Q_{h}^{\text{sfc}}, & z = -h_{\text{bot}} \end{cases}$$  \hspace{1cm} (4)$$

where $Q_{h}^{\text{sfc}}$ (W m$^{-2}$) is the sensible heat flux from the ground surface to the bottom of the vegetation canopy, and $Q_{h}^{\text{total}}$ (W m$^{-2}$) is the sensible heat flux from the top of the vegetation canopy to the height of the model forcing.

The total volumetric latent heat flux associated with evapotranspiration is defined as

$$H_{\text{lat}}^{\text{veg}} = L_{\text{sub}} E_{\text{sub}}^{\text{veg}} + L_{\text{vap}} (E_{\text{evap}}^{\text{veg}} + E_{\text{trans}}^{\text{veg}})$$ \hspace{1cm} (5)$$

where $E_{\text{sub}}^{\text{veg}}$, $E_{\text{evap}}^{\text{veg}}$ and $E_{\text{trans}}^{\text{veg}}$ (kg m$^{-3}$ s$^{-1}$) are the volumetric rates of canopy sublimation, canopy evaporation and canopy transpiration, and $L_{\text{sub}}$ and $L_{\text{vap}}$ (J kg$^{-1}$) define the latent heat of sublimation and vaporization.

2.2 Hydrology

The conservation equations for hydrology are [Clark et al., 2015b]

$$\frac{\partial \Theta_{\text{m}}^{\text{veg}}}{\partial t} = -\frac{\partial q_{\text{liq},z}^{\text{veg}}}{\partial z} - \frac{\partial q_{\text{ice},z}^{\text{veg}}}{\partial z} + \frac{E_{\text{evap}}^{\text{veg}} + E_{\text{trans}}^{\text{veg}}}{\rho_{\text{liq}}}$$ \hspace{1cm} (6)$$

$$\frac{\partial \Theta_{\text{m}}^{\text{snow}}}{\partial t} = -\frac{\partial q_{\text{liq},z}^{\text{snow}}}{\partial z} - \frac{\partial q_{\text{ice},z}^{\text{snow}}}{\partial z} + \frac{E_{\text{evap}}^{\text{snow}} + E_{\text{trans}}^{\text{snow}}}{\rho_{\text{liq}}}$$ \hspace{1cm} (7)$$

$$\frac{\partial \Theta_{\text{m}}^{\text{soil}}}{\partial t} = \frac{\partial q_{\text{liq},x}^{\text{soil}}}{\partial x} + \frac{\partial q_{\text{liq},y}^{\text{soil}}}{\partial y} + \frac{\partial q_{\text{liq},z}^{\text{soil}}}{\partial z} + \frac{E_{\text{evap}}^{\text{soil}} + E_{\text{trans}}^{\text{soil}}}{\rho_{\text{liq}}}$$ \hspace{1cm} (8)$$

where $\Theta_{\text{m}}^{\text{veg}}$ (-) is the total equivalent liquid water content, given for vegetation and snow as

$\Theta_{\text{m}}^{\text{veg}} = \Theta_{\text{liq}}^{\text{veg}} + \rho_{\text{ice}} \Theta_{\text{ice}}^{\text{veg}} / \rho_{\text{liq}}$ and $\Theta_{\text{m}}^{\text{snow}} = \Theta_{\text{liq}}^{\text{snow}} + \rho_{\text{ice}} \Theta_{\text{ice}}^{\text{snow}} / \rho_{\text{liq}}$; for soils we assume that $\rho_{\text{ice}} = \rho_{\text{liq}}$, \hspace{1cm} (9)$$
meaning that there is no volume expansion during freezing [Dall’Amico et al., 2011], and hence
\[ \Theta_m^{\text{soil}} = \Theta_{\text{liq}}^{\text{soil}} + \Theta_{\text{ice}}^{\text{soil}}. \]

On the RHS of equations (6) through (8), \( q_{\text{liq}}^{\text{veg}} \) and \( q_{\text{ice}}^{\text{veg}} \) (m s\(^{-1}\)) are the vertical fluxes of liquid water (thoughfall and canopy drainage) and ice (throughfall and unloading of snow from the canopy), \( q_{\text{liq},z}^{\text{snow}} \) (m s\(^{-1}\)) is the vertical flux of liquid water through the snowpack, \( q_{\text{ice},z}^{\text{snow}} \) (m s\(^{-1}\)) is the vertical flux of water in solid form, defined as
\[
q_{\text{ice}}^{\text{snow}} = \begin{cases} 
q_{\text{tf},\text{snow}}^{\text{snow}} + q_{\text{unload}}^{\text{snow}} & z = -h_{\text{sf}} \\
0 & z > -h_{\text{sf}} 
\end{cases}
\tag{9}
\]
where \( q_{\text{tf},\text{snow}}^{\text{snow}} \) and \( q_{\text{unload}}^{\text{snow}} \) (m s\(^{-1}\)) define the throughfall and unloading fluxes (throughfall is equal to snowfall over bare ground and at times when the canopy is completely covered with snow), and \( q_{\text{liq},x}^{\text{soil}} \), \( q_{\text{liq},y}^{\text{soil}} \), and \( q_{\text{liq},z}^{\text{soil}} \) (m s\(^{-1}\)) are the liquid fluxes through the soil matrix in the \( x \), \( y \), and \( z \) directions. Note in equation (9) that the solid precipitation flux occurs only at the top of the snowpack (\( z = -h_{\text{sf}} \)), and note in equation (8) that there is no vertical flux of ice. The evaporative losses on the RHS of equations (6) through (8) include the volumetric evaporation and sublimation fluxes from the vegetation canopy, \( E_{\text{evap}}^{\text{veg}} \) and \( E_{\text{sub}}^{\text{veg}} \) (kg m\(^{-3}\) s\(^{-1}\)), the volumetric evaporation and sublimation fluxes from the snowpack, terms \( E_{\text{evap}}^{\text{snow}} \) and \( E_{\text{sub}}^{\text{snow}} \) (kg m\(^{-3}\) s\(^{-1}\)), and the losses of water from the soil matrix due to soil evaporation and transpiration respectively, \( E_{\text{evap}}^{\text{soil}} \) and \( E_{\text{trans}}^{\text{soil}} \) (kg m\(^{-3}\) s\(^{-1}\)).

The conservation equation for the subterranean aquifer is
\[
\frac{dS_{\text{aq}}}{dt} = q_{\text{drain}}^{\text{aq}} + q_{\text{trans}}^{\text{aq}} - q_{\text{base}}^{\text{aq}} \tag{10}
\]
where \( S_{\text{aq}} \) (m) is the water storage in the aquifer, \( q_{\text{drain}}^{\text{aq}} \) (m s\(^{-1}\)) is the drainage from the bottom of the soil profile, \( q_{\text{trans}}^{\text{aq}} \) (m s\(^{-1}\)) is the transpiration loss from the aquifer (recall that fluxes are defined as positive downward), and \( q_{\text{base}}^{\text{aq}} \) (m s\(^{-1}\)) is baseflow from the aquifer to the stream.
2.3 Phase change

2.3.1 Melt-freeze

All fluxes on the RHS of equations (1) through (3) can be calculated as functions of temperature in the relevant parts of the model domain. However, the LHS of equations (2) and (3) include two state variables (the temperature $T$ and the volumetric ice content $\theta_{ice}$), and, in order to close the equations, an additional equation is needed to relate $T$ to $\theta_{ice}$. In this paper we use the differentiable functions

$$\theta_{liq}^{veg} = f(T, \Theta_m)$$  \hspace{1cm} (11)

$$\theta_{liq}^{snow} = f(T, \Theta_m)$$  \hspace{1cm} (12)

$$\theta_{liq}^{soil} = f(T, \psi_0)$$  \hspace{1cm} (13)

where ice, $\Theta_m$ (-) is total equivalent liquid water content, i.e., $\Theta_m = \theta_{liq} + \rho_{ice} \theta_{ice} / \rho_{liq}$, and $\psi_0$ (m) is the matric potential corresponding to the total water content (liquid and ice). Equations (11) through (13) allow for a fraction of liquid water at sub-freezing temperatures (i.e., supercooled liquid water). Note that $\theta_{liq}$ and $\theta_{ice}$ are interchangeable, as reductions in $\theta_{ice}$ are accompanied by corresponding increases in $\theta_{liq}$, i.e., $\rho_{liq} L_f \frac{\partial \theta_{liq}}{\partial t} = -\rho_{ice} L_f \frac{\partial \theta_{ice}}{\partial t}$. In our implementation, we make the choice to express equations (11) through (13) in terms of $\theta_{liq}$ rather than $\theta_{ice}$. This choice is made because many frozen soil models use the same soil moisture characteristics functions for freezing and thawing as for wetting and drying, where a linear relationship between $T$ and matric head $\psi$ is incorporated within the non-linear soil moisture characteristics function relating $\theta_{liq}$ and $\psi$ [Spaans and Baker, 1996; Cox et al., 1999; Niu and Yang, 2006; Dall’Amico et al., 2011].

The relationships between $\theta_{liq}$ and $T$ for vegetation and snow and the relationships between $\theta_{liq}$ and $T$ for soil are described in Appendix A.

2.3.2 Evaporation and sublimation

Phase changes from liquid water to vapor (evaporation/condensation) and ice to vapor (sublimation/frost) are included in the latent heat flux terms in the thermodynamics calculations, and are coupled with the hydrology calculations through operator splitting approximations and alternating between the sub-
matrices for thermodynamics and hydrology as part of the Newton-Raphson iterations (see Clark et al. [2015b]).

3. Flux parameterizations

3.1 Thermodynamics

3.1.1 Transmission of shortwave radiation through the vegetation canopy

An important source of predictive differences among hydrologic and land-surface models is the method used to simulate the transmission and attenuation of shortwave radiation through the vegetation canopy. The main inter-model differences stem from (i) the methods used to simulate radiation transmission through homogenous vegetation [Dickinson, 1983; Sellers, 1985; Nijssen and Lettenmaier, 1999; Mahat and Tarboton, 2012]; (ii) the methods used to parameterize the impact of the canopy gap fraction on grid-average shortwave radiation fluxes [Cescatti, 1997; Kucharik et al., 1999; Niu and Yang, 2004; Essery et al., 2008]; and (iii) the methods used to represent spatial variability in vegetation type [Koster and Suarez, 1992; Bonan et al., 2002]. In this paper the parameterizations of canopy shortwave radiation are restricted to radiation transmission through homogenous vegetation, as this approach is used in many existing models. Recent advances in modeling the impact of canopy heterogeneity on grid average fluxes [e.g., Essery et al., 2008] are not included at this stage in model development, and will be considered in future work.

The methods considered for radiation transmission through homogenous vegetation allow for different levels of model complexity. At the simplest level we include an application of Beer’s Law for direct-beam radiation (e.g., as used in VIC). At a more complex level, we include methods that model direct and diffuse beams separately and account for multiple scattering and multiple reflections [Nijssen and Lettenmaier, 1999]. Building additional complexity, we also include options for two-stream radiative transfer models as implemented in UEB [Mahat and Tarboton, 2012] and the Noah-MP model and CLM [Dickinson, 1983; Sellers, 1985; Oleson et al., 2010; Niu et al., 2011].

3.1.1.1 Beer’s Law

The interception of direct-beam shortwave radiation by vegetation at zenith angle $\theta_{zen}$ can be described as [e.g., Mahat and Tarboton, 2012]

$$\frac{dQ_{wb}}{dz} = -Q_{wb} \frac{G\eta}{\cos(\theta_{zen})}$$  
(14)
where $Q_{swb}$ (W m$^{-2}$) is the downward shortwave radiation flux for the direct-beam, $G$ (m$^2$ m$^{-2}$) is the leaf orientation factor defining the average area of leaves when viewed from direction $\theta_{zen}$, $\eta_l$ (m$^2$ m$^{-3}$) is the leaf+stem density and $z$ (m) is the vertical coordinate, positive downwards.

Given the canopy depth $D_{can} = h^\text{top}_\text{can} - h^\text{bot}_\text{can}$ (m), integrating equation (14) results in Beer’s law [Mahat and Tarboton, 2012]

$$Q_{swb} (D_{can}) = Q_{swb}^0 \exp \left(-G \eta_l D_{can} \frac{\cos(\theta_{zen})}{\cos(\theta_{zen})}\right)$$

where $Q_{swb}^0$ (W m$^{-2}$) defines the direct-beam shortwave radiation flux at the top of the canopy.

The penetration of direct beam shortwave radiation through the canopy in the absence of scattering can then be described as

$$\tau_{pb} = \frac{Q_{swb} (D_{can})}{Q_{swb}^0} = \exp \left(-G \frac{V_{ex}}{\cos(\theta_{zen})}\right)$$

where $V_{ex} = \eta_l D_{can}$ (m$^2$ m$^{-2}$) defines the total exposed leaf+stem area, that is, the leaf+stem area that is not buried by snow.

Under the simplifying assumption that the total transmission of direct and diffuse radiation ($\tau$) is equal to the penetration of direct beam radiation, i.e.,

$$\tau = \tau_{pb}$$

the shortwave radiation flux absorbed by the vegetation canopy and the ground surface is

$$Q_{\text{sw net}}^\text{veg} = Q_{sw}^0 (1-\tau) (1-\alpha^\text{veg}) + Q_{sw}^0 \tau \alpha^\text{fc} (1-\tau)$$

$$Q_{\text{sw net}}^\text{fc} = Q_{sw}^0 \tau (1-\alpha^\text{fc})$$

where $\alpha^\text{veg}$ and $\alpha^\text{fc}$ (-) define the albedo of the vegetation canopy and ground surface respectively, and $Q_{sw}^0 = Q_{swb}^0 + Q_{swd}^0$ (W m$^{-2}$) is the sum of direct-beam and diffuse radiation at the top of the canopy. Note that $\alpha^\text{veg}$ (-) defines the bulk canopy albedo, which is typically much lower than the reflectance of an individual leaf due to the partial trapping of light by layers of leaves [Dickinson, 1983; Nijssen and Lettenmaier, 1999].
The first term on the right-hand-side of equation (18) represents the shortwave radiation absorbed by the canopy on the downward pass, while the second term represents the shortwave radiation that is reflected by the surface and absorbed on the upward pass. The approach described by equations (17) through (19) is similar to that used in the Variable Infiltration Capacity (VIC) model [Liang et al., 1994] and the Distributed Hydrology Soil Vegetation Model (DHSVM) [Wigmosta et al., 1994; Wigmosta and Lettenmaier, 1999], except that (i) VIC and DHSVM do not include a dependence on the solar zenith angle; and (ii) VIC assumes that all radiation reflected by the ground surface is lost through the top of the canopy, i.e., VIC does not include the second term in equation (18), which can be quite important when the ground surface is covered by snow.

3.1.1.2 Accounting for diffuse radiation, scattering and multiple reflections

The approach described above assumes all radiation is from a single direct beam and ignores the effects of scattering and multiple reflections. Nijssen and Lettenmaier [1999] showed that the penetration of diffuse radiation, \( \tau_{pd} \), can be obtained by integrating over the upper hemisphere, as

\[
\tau_{pd} = \frac{1}{Q_{swd}^0} \int Q_{swd}^0 (\theta_{zen}) \tau_{pb} (\theta_{zen}) \cos(\theta_{zen}) \, d\Omega \tag{20}
\]

where \( Q_{swd}^0 (\theta_{zen}) \) is the above-canopy diffuse-beam radiation from the direction \( \theta_{zen} \), and \( \tau_{pb} (\theta_{zen}) \) is the direct beam transmissivity from the direction \( \theta_{zen} \) as defined in equation (16).

Assuming diffuse radiation is isotropic, a solution to equation (20) is [Nijssen and Lettenmaier, 1999; Mahat and Tarboton, 2012]

\[
\tau_{pd} = \frac{Q_{swd} (D_{can})}{Q_{swd}^0} = (1 - GV_{ex}) \exp(-GV_{ex}) + (GV_{ex})^2 E_i(1, GV_{ex}) \tag{21}
\]

where \( E_i(n,x) \) is the exponential integral, and recall that \( V_{ex} = \eta_l D_{can} \) defines the total exposed leaf+stem area (m\(^2\) m\(^{-2}\)), that is, the leaf+stem area that is not buried by snow.

The total penetration of radiation through the canopy is then

\[
\tau_{pbd} = \frac{Q_{swb}^0}{Q_{sw}^0} \tau_{pb} + \frac{Q_{swd}^0}{Q_{sw}^0} \tau_{pd} \tag{22}
\]

where \( \tau_{pb} \) is given by the solution to Beer’s law in equation (16).
Nijssen and Lettenmaier [1999] introduce a simple method to account for scattering and multiple reflections, providing total transmission $\tau$ as

$$\tau = \left( \frac{1}{\varphi \tau_{pd}} \right)^{\phi_s} \quad (23)$$

where the exponent $\phi_s$ accounts for scattering of radiation within the vegetation canopy. The value of $\phi_s$ should generally fall in the range 0.7 to 0.85, and $\phi_s = 0.81$ in the simulations presented by Nijssen and Lettenmaier [1999].

Given equation (23) and the assumption that all shortwave radiation reflected by the ground surface is diffuse, the shortwave radiation flux absorbed by the vegetation canopy and the ground surface can be given as

$$Q_{\text{swnet}}^\text{veg} = Q_{\text{sw}}^0 (1 - \tau)(1 - \alpha_{\text{veg}}) + Q_{\text{sw}}^0 m_r \tau \alpha_{\text{veg}}^\phi \left(1 - \left(\frac{1}{\varphi \tau_{pd}} \right)^{\phi_s}\right) \quad (24)$$

$$Q_{\text{swnet}}^\text{sfc} = Q_{\text{sw}}^0 m_r \tau (1 - \alpha_{\text{sfc}}) \quad (25)$$

where the terms on the right-hand-side of equation (24) describe the downward and upward radiation absorbed by the vegetation canopy, respectively, and $m_r$ accounts for multiple reflections between the ground and canopy.

Note that equation (24) assumes that all reflected radiation is diffuse, and reflection of upward radiation from the canopy is incorporated in the calculation of $\tau$ in equation (23) through the multiple reflection factor $m_r$.

The multiple reflection factor $m_r$ is derived from the infinite series of reflections between the surface and vegetation [Nijssen and Lettenmaier, 1999] as

$$m_r = \sum_{i=0}^{\infty} \left[ \alpha_{\text{veg}}^\phi \alpha_{\text{veg}}^\phi \right] = \frac{1}{1 - \alpha_{\text{sfc}}^\phi \alpha_{\text{veg}}^\phi \left(1 - \left(\frac{1}{\varphi \tau_{pd}} \right)^{\phi_s}\right)} \quad (26)$$

with the factor $\left(1 - \left(\frac{1}{\varphi \tau_{pd}} \right)^{\phi_s}\right)$ included to represent the fraction of radiation transmitted and scattered upwards through the canopy [see also Mahat and Tarboton, 2012].

3.1.1.3 Two-stream radiative transfer models
**Mahat and Tarboton** [2012] describe a two stream radiative transfer model based on the assumptions that radiation is scattered equally in an upward and downward direction and that scattering is along the same path as the incoming radiation. These assumptions yield

\[
- \frac{dU_{sw}}{dz} = \frac{G\eta_l}{\cos(\theta_{zen})} \left[ -U_{sw} + \alpha_{leaf} \frac{U_{sw} + Q_{sw}}{2} \right]
\]  

(27)

\[
\frac{dQ_{sw}}{dz} = \frac{G\eta_l}{\cos(\theta_{zen})} \left[ -Q_{sw} + \alpha_{leaf} \frac{U_{sw} + Q_{sw}}{2} \right]
\]  

(28)

where \(\alpha_{leaf}\) is the leaf scattering coefficient, distinguished from the bulk canopy albedo used in equation (18) and (24), and \(U_{sw}\) and \(Q_{sw}\) (W m\(^{-2}\)) are the intensity of the upward and downward beams.

The solution for transmission over an infinitely deep canopy is [**Mahat and Tarboton**, 2012]

\[
\tau_{deep,b} = \frac{Q_{swb}(D_{can})}{Q_{swb}^0} = \exp \left( -k' G \frac{V_{ex}}{\cos(\theta_{zen})} \right)
\]  

(29)

\[
\tau_{deep,d} = \frac{Q_{swd}(D_{can})}{Q_{swd}^0} = (1 - k' G V_{ex}) \exp \left( -k' G V_{ex} \right) + \left( k' G V_{ex} \right)^2 E_i(1, k' G V_{ex})
\]  

(30)

where \(k' = \sqrt{1 - \alpha_{leaf}}\). Equations (29) and (30) are similar to the expressions in equations (16) and (21), except the factor \(k'\) is included to account for the effects of multiple scattering. Recall that \(V_{ex} = \eta_l D_{can}\).

The upward reflection factor \(\alpha_{deep}^{vg}\) giving the fraction of radiation reflected from a deep canopy with multiple scattering is

\[
\alpha_{deep}^{vg} = \frac{U_{sw}(z)}{Q_{sw}(z)} = \frac{1 - k'}{1 + k'}
\]  

(31)

**Mahat and Tarboton** [2012] use the principle of superposition to derive solutions for a finite canopy, as

\[
\tau_b = \frac{Q_{swb}(D_{can})}{Q_{swb}^0} = \frac{\tau_{deep,b} \left[ 1 - \left( \alpha_{deep}^{vg} \right)^2 \right]}{1 - \left( \alpha_{deep}^{vg} \right)^2 (\tau_{deep,b})^2}
\]  

(32)

\[
\alpha_b^{vg} = \frac{U_{swb}^0}{Q_{swb}^0} = \frac{\alpha_{deep}^{vg} \left[ 1 - (\tau_{deep,b})^2 \right]}{1 - \left( \alpha_{deep}^{vg} \right)^2 (\tau_{deep,b})^2}
\]  

(33)
where $\tau_b$ and $\alpha_{b}^{\text{veg}}$ define the transmittance and reflectance for direct-beam radiation. Equations (32) and (33) can also be used to obtain $\tau_d$ and $\alpha_{d}^{\text{veg}}$ by replacing the direct transmittance $\tau_{\text{deep,b}}$ with the diffuse transmittance $\tau_{\text{deep,d}}$ [Mahat and Tarboton, 2012].

The shortwave radiation flux absorbed by the vegetation canopy and the ground surface can now be given as (now distinguishing between direct and diffuse surface albedo, $\alpha_{b}^{\text{sfc}}$ and $\alpha_{d}^{\text{sfc}}$, as defined in the next section),

$$Q_{\text{swnet}}^{\text{veg}} = \left[ Q_{\text{swnb}}^{0} (1 - \tau_{b})(1 - \alpha_{b}^{\text{veg}}) + Q_{\text{swnwd}}^{0} (1 - \tau_{d})(1 - \alpha_{d}^{\text{veg}}) \right]$$

$$+ \left( Q_{\text{swnb}}^{0} \tau_{b} \alpha_{b}^{\text{sfc}} + Q_{\text{swnwd}}^{0} \tau_{d} \alpha_{d}^{\text{sfc}} \right) m_r (1 - \tau_{d})$$

$$Q_{\text{swnet}}^{\text{sfc}} = \left[ Q_{\text{swnb}}^{0} \tau_{b} (1 - \alpha_{b}^{\text{sfc}}) + Q_{\text{swnwd}}^{0} \tau_{d} (1 - \alpha_{d}^{\text{sfc}}) \right] m_r$$

where $m_r$ is the multiple reflections factor as defined in equation (26) but computed using diffuse reflectances and defined as $m_r = \left[ 1 - \alpha_{d}^{\text{veg}} \alpha_{d}^{\text{sfc}} (1 - \tau_{d}) \right]^{-1}$. As in equation (24) it is assumed that all upward radiation is diffuse.

We also consider the two-stream radiative transfer model of Dickinson [1983] and Sellers [1985], as implemented in both the Community Land Model [Oleson et al., 2010] and the Noah-MP model [Niu et al., 2011]. In this approach fluxes are computed separately for visible and near-infra-red wavelengths. Complete algorithmic details are provided by Oleson et al. [2010] and are not repeated here.

### 3.1.2 Surface albedo

The surface albedo has a large impact on the surface energy balance, and on the melt rate when snow is present. The surface albedo ($\alpha_{\text{sfc}}$) depends on the fractional snow covered area, as

$$\alpha_{\text{sfc}} = \left(1 - f_{\text{snow}}^{\text{sfc}} \right) \alpha_{\text{soil}}^{\text{sfc}} + f_{\text{snow}}^{\text{sfc}} \alpha_{\text{snow}}^{\text{sfc}}$$

where $\alpha_{\text{soil}}^{\text{sfc}}$ and $\alpha_{\text{snow}}^{\text{sfc}}$ are the albedo of soil and snow surface respectively, and $f_{\text{snow}}^{\text{sfc}}$ is the fraction of ground covered in snow, currently represented as a step function with $f_{\text{snow}}^{\text{sfc}} = 1$ if $h_{\text{snow}} > h_{\text{cov}}$ and $f_{\text{snow}}^{\text{sfc}} = 0$ otherwise, where $h_{\text{snow}}$ (m) is the depth of the snowpack and $h_{\text{cov}}$ (m) is the maximum depth required for full snow coverage. Equation (36) is used for direct and diffuse albedo, and for the albedo in visible and near infra-red wavelengths. Snow albedo is a model state variable; see Clark et al. [2015b].
3.1.2.1 Temporal decay in snow albedo

An important source of differences in predictive behavior in the representation of thermodynamics arises from the methods used to parameterize snow albedo [Essery et al., 2013]. Different model representations of snow albedo include (1) semi-empirical parameterizations to describe the temporal decay in snow albedo after snowfall events [e.g., Verseghy, 1991; Yang et al., 1997], and (2) more physically realistic approaches that (i) explicitly simulate the growth of snow grains [e.g., Wiscombe and Warren, 1980; Jordan, 1991; Tribbeck et al., 2006], and (ii) explicitly simulate the deposition, burial, and re-emergence of atmospheric aerosols (e.g., soot, dust) and forest litter, and the impacts of soot, dust and forest litter on absorption of solar radiation at different depths in the snowpack [e.g., Warren and Wiscombe, 1980; Hardy et al., 2000; Flanner et al., 2007].

In this work we follow the approaches used in Niu et al. [2011] and restrict attention to two widely used semi-empirical albedo parameterizations – the Canadian Land Surface Scheme (CLASS) described by Verseghy [1991], where the albedo decay rate is fixed in time, and the Biosphere-Atmosphere Transfer Scheme (BATS) described by Yang et al. [1997], where the albedo decay rate varies over time. A parameterization similar to CLASS is used in VIC [Andreadis et al., 2009], and the BATS albedo parameterization is used in CLM3.0 and UEBveg.

The CLASS approach uses two “decay curves”, one for accumulation periods and the other for periods of melting. This is implemented by setting \( \alpha_{\text{snow}} \) if no melting occurs in the time step, and setting \( \alpha_{\text{snow}} = 0.5 \) if melting occurs. The \( \kappa_\alpha \) parameter in CLASS is temporally constant and set to 0.01/3600 s\(^{-1}\) [Verseghy, 1991]. This approach assumes that the direct beam albedo is identical to the diffuse albedo, which is justified because the difference between direct and diffuse albedos is only pronounced at high solar zenith angles when shortwave radiation fluxes are small.

The BATS approach simulates the albedo separately for visible and near-infrared wavelengths (where, for example, as detailed in Yang et al. [1997], \( \alpha_{\text{max,d}} \), \( \alpha_{\text{min,d}} \) are set to [0.85, 0.75] and [0.65, 0.15] for the visible and near-infrared wavelengths, respectively), and the albedo decay rate \( \kappa_\alpha \) depends on snow properties as

\[
\kappa_\alpha = \kappa_{\alpha,0} \left( r_1 + r_2 + r_3 \right)
\]

(37)

where \( \kappa_{\alpha,0} \) (s\(^{-1}\)) is the time delay scaling factor (\( \kappa_{\alpha,0} = 10^{-6} \) in BATS), and \( r_1 \), \( r_2 \), and \( r_3 \) are aging factors, with
\[ r_1 = \exp \left[ -\chi_{\text{temp}} (T_{frz} - T_{sfc}) \right] \quad (38) \]
\[ r_2 = (r_1)^{10} \quad (39) \]

where \( r_1 \) represents effects of grain growth due to vapor diffusion, \( r_2 \) represents the additional effects of grain growth when the snow temperature is near the freezing point \( T_{frz} \), and \( r_3 \) is an adjustable parameter representing effects of dirt and soot (in BATS \( r_3 = 0.01 \) over Antarctica and \( r_3 = 0.3 \) elsewhere). In contrast to CLASS, where the direct-beam albedo is assumed equal to the diffuse albedo, the direct-beam albedo in BATS is set to the diffuse albedo plus an additive factor at high solar zenith angles [Yang et al., 1997].

In the formulation in equation (38), \( r_1 \) differs slightly from the function presented in Yang et al. [1997], and is defined to be identical to the function used for snow compaction described by Anderson [1976], where \( \chi_{\text{temp}} \) is a scaling factor (K\(^{-1}\)), and \( T_{frz} = 273.16 \) and \( T_{sfc} \) are respectively the freezing point of pure water (K) and the temperature of the snow surface (K), taken as the temperature of the upper-most snow layer. The \( r_1 \) function defined here provides almost identical results to the \( r_1 \) function defined in Yang et al. [1997] when \( \chi_{\text{temp}} = 0.07 \) (important to mimic the behavior of the BATS parameterization), and provides more consistency in different parts of the model, as the same parameterization is used to estimate the grain growth due to vapor diffusion in both the snow albedo and snow compaction routines.

### 3.1.3 Canopy longwave radiation fluxes

Longwave radiation is handled similarly across land-surface models. There are three sources of longwave radiation [Mahat and Tarboton, 2012]: the longwave flux from the sky \((Q_{lw}^{sk})\), i.e., the model forcing at the upper boundary, and the longwave fluxes from the vegetation \((Q_{lw}^{veg})\) and the surface \((Q_{lw}^{sfc})\), defined as

\[ Q_{lw}^{veg} = \varepsilon_{veg} \sigma_{sb} (T_{veg}^4) \quad (40) \]
\[ Q_{lw}^{sfc} = \varepsilon_{sfc} \sigma_{sb} (T_{sfc}^4) \quad (41) \]

where \( \sigma_{sb} \) (W m\(^{-2}\) K\(^{-4}\)) is the Stefan-Boltzmann constant, \( \varepsilon_{veg} \) and \( \varepsilon_{sfc} \) (-) define the emissivity from the vegetation and the surface respectively, with \( \varepsilon_{veg} \) expressed alternatively as \( \varepsilon_{veg} = 1 - \exp(-V_{es}) \) (e.g.,
as in Oleson et al., [2010]) or \( \varepsilon^\text{veg} = 1 - \tau_d \) [Mahat and Tarboton, 2012], where \( \tau_d \) is the transmissivity of diffuse shortwave radiation obtained in the shortwave radiation parameterizations, and \( \varepsilon^\text{sfc} = 0.98 \) for vegetated surfaces but can be adjusted for different land cover types (note that absorption is equal to emissivity).

Longwave radiation from the sky can be partitioned as [Mahat and Tarboton, 2012]

\[
Q^\text{sky}^\rightarrow\text{veg} = Q^\text{sky}^\rightarrow\text{veg} + Q^\text{sky}^\rightarrow\text{sfc}\left(1 - \varepsilon^\text{veg}\right)\left(1 - \varepsilon^\text{sfc}\right)\varepsilon^\text{veg} \tag{42}
\]

\[
Q^\text{sky}^\rightarrow\text{sfc} = Q^\text{sky}^\rightarrow\text{sfc}\left(1 - \varepsilon^\text{veg}\right)\varepsilon^\text{sfc} \tag{43}
\]

\[
Q^\text{sky}^\rightarrow\text{sky} = Q^\text{sky}^\rightarrow\text{sky}\left(1 - \varepsilon^\text{veg}\right)\left(1 - \varepsilon^\text{sfc}\right)\left(1 - \varepsilon^\text{veg}\right) \tag{44}
\]

where \( Q^\text{sky}^\rightarrow\text{veg} \) and \( Q^\text{sky}^\rightarrow\text{sfc} \) represent the longwave radiation from the upper boundary absorbed by vegetation and the surface respectively, and \( Q^\text{sky}^\rightarrow\text{sky} \) represents the longwave radiation transmitted through the vegetation \( (1 - \varepsilon^\text{veg}) \), reflected by the surface \( (1 - \varepsilon^\text{sfc}) \), and then transmitted through the vegetation again to the upper boundary \( (1 - \varepsilon^\text{veg}) \). The second term in equation (42) represents the longwave radiation transmitted through the vegetation \( (1 - \varepsilon^\text{veg}) \), reflected by the surface \( (1 - \varepsilon^\text{sfc}) \), and absorbed by the vegetation \( (\varepsilon^\text{veg}) \).

Longwave radiation emitted from vegetation can be partitioned as

\[
Q^\text{veg}^\rightarrow\text{sky} = Q^\text{veg}^\rightarrow\text{sky} + Q^\text{veg}^\rightarrow\text{veg}\left(1 - \varepsilon^\text{sfc}\right)\left(1 - \varepsilon^\text{veg}\right) \tag{45}
\]

\[
Q^\text{veg}^\rightarrow\text{sfc} = Q^\text{veg}^\rightarrow\text{sfc}\varepsilon^\text{sfc} \tag{46}
\]

\[
Q^\text{veg}^\rightarrow\text{veg} = Q^\text{veg}^\rightarrow\text{veg}\left(1 - \varepsilon^\text{sfc}\right)\varepsilon^\text{veg} \tag{47}
\]

where \( Q^\text{veg}^\rightarrow\text{sky} \) is the longwave radiation emitted from the vegetation that escapes through the upper boundary, with the second term in equation (45) defining radiation reflected by the surface and transmitted upwards through the canopy to the upper boundary. Some of the longwave flux emitted from the vegetation is absorbed by the surface \( (Q^\text{veg}^\rightarrow\text{sfc}) \), and the remainder is reflected. The portion that is not transmitted upwards through the canopy is re-absorbed by the vegetation \( (Q^\text{veg}^\rightarrow\text{veg}) \). Note that

\[
Q^\text{veg}^\rightarrow\text{sky} + Q^\text{veg}^\rightarrow\text{sfc} + Q^\text{veg}^\rightarrow\text{veg} = 2Q^\text{veg} \] since the canopy radiates both upward and downward.
Finally, longwave radiation emitted from the surface is partitioned as

\[ Q_{lw}^{sfc \rightarrow sky} = Q_{lw}^{sfc} (1 - e^{veg}) \]  
(48)

\[ Q_{lw}^{sfc \rightarrow veg} = Q_{lw}^{sfc} e^{veg} \]  
(49)

where the longwave radiation emitted from the surface is either transmitted upward through the canopy to the upper boundary (\( Q_{lw}^{sfc \rightarrow sky} \)) or absorbed by vegetation (\( Q_{lw}^{sfc \rightarrow veg} \)).

The net longwave radiation at the vegetation and the surface can now be defined as

\[ Q_{lw}^{net} = Q_{lw}^{sky \rightarrow veg} + Q_{lw}^{sfc \rightarrow veg} + Q_{lw}^{veg \rightarrow veg} - 2Q_{lw}^{veg} \]  
(50)

\[ Q_{lw}^{net} = Q_{lw}^{sky \rightarrow sfc} + Q_{lw}^{veg \rightarrow sfc} - Q_{lw}^{sfc} \]  
(51)

where the factor of 2 in equation (50) accounts for the longwave radiation emitted at the top and the bottom of the canopy.

### 3.1.4 Within- and below-canopy turbulence

We explicitly simulate the turbulent fluxes within and below the vegetation canopy, including fluxes from the ground surface to the canopy air space, from the vegetation canopy to the canopy air space, and from the canopy air space to the forcing level above the canopy. The development that follows assumes a single canopy layer, obtained through spatial discretization of equations (2) and (1), as detailed in Clark et al. [2015b]. Note that the fluxes are defined per unit area, and relate to the volumetric fluxes in equations (2) and (1) as

\[ Q_{v}^{veg} = \frac{D_{can}}{H_{sen}^{veg}}, \quad Q_{v}^{evap} = \frac{D_{can}}{L_{vap}^{evap} E_{evap}^{veg}}, \quad \text{and} \quad Q_{v}^{trans} = \frac{D_{can}}{L_{vap}^{trans} E_{trans}^{veg}}. \]

The approach used here to simulate within- and below- canopy turbulence, as introduced by Choudhury and Monteith [1988], is shared across existing land-surface models. Key recent examples are CLM [Oleson et al., 2010], Noah-MP [Niu et al., 2011], and UEBveg [Mahat et al., 2013]. Key modeling decisions have to do with: (i) whether the models include a state variable for canopy air space temperature, as recommended by Vidale and Stockli [2005] and as implemented in SUMMA; and (ii) parameterizations of details in the canopy wind profile and resistance terms (including estimation of the roughness length, and atmospheric stability corrections).

#### 3.1.4.1 Sensible and latent heat fluxes from the surface and vegetation

The sensible and latent heat fluxes from the vegetation and the land (or snow) surface are defined following Niu et al. [2011] as
\[
Q_{h}^{\text{veg}} = -\rho_{\text{air}} c_{p} C_{h}^{\text{veg}} \left( T_{\text{veg}} - T_{\text{cas}} \right) 
\]
\[
Q_{h}^{\text{fc}} = -\rho_{\text{air}} c_{p} C_{h}^{\text{fsc}} \left( T_{\text{fsc}} - T_{\text{cas}} \right) 
\]
\[
Q_{\text{evap}}^{\text{veg}} = -\frac{L_{\text{vap}} \rho_{\text{air}} e}{P_{\text{air}}} C_{\text{evap}}^{\text{veg}} \left[ e_{\text{sat}} \left( T_{\text{veg}} \right) - e_{\text{cas}} \right] 
\]
\[
Q_{\text{trans}}^{\text{veg}} = -\frac{L_{\text{vap}} \rho_{\text{air}} e}{P_{\text{air}}} C_{\text{trans}}^{\text{veg}} \left[ e_{\text{sat}} \left( T_{\text{veg}} \right) - e_{\text{cas}} \right] 
\]
\[
Q_{l}^{\text{fc}} = -\frac{L_{\text{vap}} \rho_{\text{air}} e}{P_{\text{air}}} C_{\text{fsc}}^{\text{fc}} \left[ \phi_{\text{hum}} e_{\text{sat}} \left( T_{\text{fsc}} \right) - e_{\text{cas}} \right] 
\]

where \(\rho_{\text{air}}\) (kg m\(^{-3}\)) and \(c_{p}\) (J kg\(^{-1}\) K\(^{-1}\)) are the density and heat capacity of air, \(L_{\text{vap}}\) (J kg\(^{-1}\)) is the latent heat of vaporization, which is set to the latent heat of sublimation in equation (54) if the vegetation or surface temperature is less than the freezing point of pure water, \(e \approx 0.622\) is the ratio of the molecular weight of water vapor to dry air, \(P_{\text{air}}\) (Pa) is the air pressure, \(\phi_{\text{hum}}\) (-) is the relative humidity of air in the surface soil (or snow) pore space, \(C_{\text{xx}}\) (m s\(^{-1}\)) defines the conductance of heat and water vapor, \(T\) (K) is temperature, \(e\) (Pa) is the vapor pressure and \(e_{\text{sat}}(T)\) (Pa) is the saturated vapor pressure at temperature \(T\), and the superscripts “veg”, “sfc”, and “cas” define the surface, vegetation and canopy air space respectively. Note that the fluxes in equations (53) and (56) are defined as positive towards the surface, and the fluxes in equations (52), (54), and (55) are defined as positive towards the vegetation canopy.

The total heat flux to the atmosphere (positive downward) is given as

\[
Q_{h} = -\rho_{\text{air}} c_{p} C_{h} \left( T_{\text{cas}} - T_{\text{air}} \right) 
\]
\[
Q_{l} = -\frac{L_{\text{vap}} \rho_{\text{air}} e}{P_{\text{air}}} C_{w} \left( e_{\text{cas}} - e_{\text{air}} \right) 
\]

where \(C_{h}^{\text{air}}\) and \(C_{w}^{\text{air}}\) (m s\(^{-1}\)) is the conductance of heat and water vapor from the canopy to the atmosphere, and \(T_{\text{air}}\) (K) and \(e_{\text{air}}\) (Pa) are the temperature and vapor pressure at the upper boundary (i.e., the model forcing). For the special case where no vegetation is present or the vegetation is completely buried with snow, the turbulent heat fluxes are computed using equations (57) and (58), but replacing the canopy air space temperature and vapor pressure with the surface temperature and vapor pressure and
computing stability adjustments to the heat and water vapor conductances $C^h$ and $C^w$ based on the surface temperature.

The conductance terms are defined as [e.g., Niu et al., 2011]

$$C^h_{veg} = \frac{V_{ex}}{r_l} \quad (59)$$

$$C^h_{sfc} = \frac{1}{r_g} \quad (60)$$

$$C^{veg}_{evap} = f^{veg}_{wet} \frac{V_{ex}}{r_l} \quad (61)$$

$$C^{veg}_{trans} = \left(1 - f^{veg}_{wet}\right) \left(\frac{L_{ex,sun}}{r_l + r_{s,sun}} + \frac{L_{ex,shd}}{r_l + r_{s,shd}}\right) \quad (62)$$

$$C^w_{sfc} = \frac{1}{r_g + r_{soil}} \quad (63)$$

$$C^w_{air} = C^h_{air} = \frac{1}{r_a} \quad (64)$$

where, $r_g$, $r_{soil}$, $r_l$, $r_{s,sun}$, $r_{s,shd}$ (s m⁻¹) are, respectively, the ground resistance, the soil resistance to vapor flux, the leaf resistance, and the stomatal resistance for sunlit and shaded leaves, and $V_{ex}$ (m² m⁻²) is the exposed vegetation area (stem+leaf), $L_{ex,sun}$ and $L_{ex,shd}$ (m² m⁻²) is the exposed area of sunlit and shaded leaves, and $f^{veg}_{wet}$ (-) is the fraction of the canopy that is wet.

The exposed area of sunlit and shaded leaves is given as

$$L_{ex,sun} = L_{ex} f^{veg}_{sun} \quad (65)$$

$$L_{ex,shd} = L_{ex} \left(1 - f^{veg}_{sun}\right) \quad (66)$$

where $f^{veg}_{sun}$ (-) is the fraction of sunlit leaves, defined following Sellers [1985]

$$f^{veg}_{sun} = \frac{1 - \exp\left(-\chi_{sun} V_{ex}\right)}{\chi_{sun} V_{ex}} \quad (67)$$
with $\chi_{\text{sun}}$ (-) is the optical depth of the direct beam per unit leaf area, computed as a function of leaf orientation, solar zenith angle, and leaf transmittance and reflectance.

The fraction of the canopy that is wet is parameterized following Deardorff [1978].

$$ f_{\text{wet}}^\text{veg} = \left( \frac{W_{\text{liq}}^{\text{veg}}}{I_{\text{b,liq}} V_{\text{ex}}} \right)^w $$

where $W_{\text{liq}}^{\text{veg}}$ (kg m$^{-2}$) is the storage of liquid water on the vegetation canopy and $I_{\text{b,liq}}$ (kg m$^{-2}$) is interception capacity of liquid water per unit $V_{\text{ex}}$, and the exponent $w \approx 0.67$ following Deardorff [1978].

Methods to calculate the temperature and vapor pressure of the canopy air space differ across models. Given $Q_h = Q_{h}^{\text{veg}} + Q_{h}^{\text{sfc}}$ and $Q_l = Q_{l}^{\text{veg}} + Q_{l}^{\text{sfc}}$, rearranging equations (52) through (58) provides the temperature and vapor pressure of the canopy air space as

$$ T_{\text{cas}} = \frac{C_{h}^{\text{air}} T_{\text{air}} + C_{h}^{\text{veg}} T_{\text{veg}} + C_{h}^{\text{sfc}} T_{\text{sfc}}} {C_{h}^{\text{air}} + C_{h}^{\text{veg}} + C_{h}^{\text{sfc}}} $$

$$ e_{\text{cas}} = \frac{C_{w}^{\text{air}} e_{\text{air}} + (C_{\text{evap}}^{\text{veg}} + C_{\text{trans}}^{\text{veg}}) e_{\text{sat}} (T_{\text{veg}}) + C_{w}^{\text{sfc}} \phi_{\text{hum}} e_{\text{sat}} (T_{\text{sfc}})} {C_{w}^{\text{air}} + C_{w}^{\text{evap}} + C_{w}^{\text{trans}} + C_{w}^{\text{sfc}}} $$

such that canopy air space variables are a weighted combination of state variables on the vegetation, surface, and upper boundary, where the weight assigned to each surface depends on the conductance. This approach assumes that there is no heat and vapor pressure storage in the canopy air space [Vidale and Stockli, 2005]. The numerical implementation of equations (69) and (70) is however rather cumbersome, as the canopy air space temperature depends on the conductance terms, which in turn depend on the within-canopy and above-canopy stability corrections, which in turn depend on the canopy air space temperature.

We select canopy air space temperature as a state variable [Flerchinger et al., 1998; Vidale and Stockli, 2005] and explicitly simulate the time evolution of canopy air space temperature using the conservation equation defined in Section 2.1. This simplifies the numerical solution because atmospheric stability can be computed using the temperature vector in each Newton-Raphson iteration. Vapor pressure of the canopy air space is computed using equation (70). The approach for computing eddy diffusivity within the canopy is explained below.
3.1.4.2 Wind profiles and aerodynamic resistance

In describing wind profiles and aerodynamic resistance, we depart from the positive-downward coordinate system used in the rest of the paper and use the variable $h$ (m) to define the height above the soil surface ($h$ is defined as positive-upward). This is done to maintain consistency with the presentation in previous papers.

There are substantial inter-model differences in parameterization of wind profiles through the canopy: for example, the original model of Choudhury and Monteith [1988] developed for wheat assumes an exponential reduction in wind speed through the canopy extends to the ground surface, and this parameterization has been directly applied by Niu and Yang [2004] for forests. In contrast, Mahat et al. [2013] and Andreadis et al. [2009] assume winds follow a logarithmic profile from the upper boundary to the top of the canopy, an exponential profile within the canopy, and a logarithmic profile again below the canopy.

The Mahat et al. [2013] parameterization is

$$u(h) = \begin{cases} 
\frac{1}{k} u_{\text{veg}} \ln \left( \frac{h - d}{z_0^{\text{veg}}} \right) & h \geq h_{\text{top}}^{\text{veg}} \\
\frac{1}{k} u_{\text{sfc}} \ln \left( \frac{h}{z_0^{\text{sfc}}} \right) & h < h_{\text{bot}}^{\text{veg}} 
\end{cases}$$

(71)

where $u(h)$ (m s$^{-1}$) is the wind speed at height $h$ (m), $a_w$ (-) is an exponential wind decay coefficient, $d$ (m) is the zero-plane displacement height, $u_{\Omega}^{\text{veg}}$ (m s$^{-1}$) is the friction velocity ($\Omega = \text{sfc}, \text{veg}$), $k \approx 0.4$ is von Karman’s constant, $z_0^{\text{veg}}$ and $z_0^{\text{sfc}}$ (m) define the roughness length of the vegetation canopy and the surface, and $h_{\text{top}}^{\text{veg}}$ and $h_{\text{bot}}^{\text{veg}}$ (m) define the height of the top and the bottom of the vegetation canopy.

The friction velocity $u_{\Omega}^{\text{veg}}$ is defined as

$$u_{\Omega}^{\text{veg}} = \frac{k u_{\text{air}}}{\ln \left( \frac{h_{\text{air}} - d}{z_0^{\text{veg}}} \right)}$$

(72)
and

\[
\begin{align*}
    u_{\text{wc}} = \frac{ku_{\text{bot}}^{\text{veg}}}{\ln \left( \frac{h_{\text{bot}}^{\text{veg}}}{z_0^{\text{wc}}} \right)}
\end{align*}
\]  

(73)

where \( u_{\text{air}} \) and \( u_{\text{bot}}^{\text{veg}} \) (m s\(^{-1}\)) define the wind speed at the model forcing level (\( h_{\text{air}} \)) and the height of the bottom of the vegetation (\( h_{\text{bot}}^{\text{veg}} \)).

The parameterizations of wind profiles depend critically on the exponential decay coefficient describing reduction in windspeed throughout the canopy, i.e., \( a_w \) in equation (71) is both represented as constant across all vegetation types (e.g., \( a_w = 3 \) as in Niu and Yang [2004]), or parameterized in terms of the leaf area index, canopy height, and leaf dimension as [Norman et al., 1995]

\[
a_w = a_{w,0} (V_{ex})^{2/3} \left( \frac{h_{\text{top}}^{\text{veg}}}{S_{\text{leaf}}} \right)^{1/3}
\]  

(74)

where \( a_{w,0} \approx 0.28 \) and \( S_{\text{leaf}} \) (m) is the mean leaf size given by four times the leaf area divided by the leaf perimeter. These parameterizations can have a large impact on the form of the wind profile and the turbulent fluxes within and above the canopy.

The canopy roughness length and zero-plane displacement height are also parameterized in different ways [e.g., Choudhury and Monteith, 1988; Raupach, 1994]. The Choudhury and Monteith [1988] parameterization is

\[
d = 1.1h_{\text{top}}^{\text{veg}} \ln \left[ 1 + \left( c_d V_{ex} \right)^{1/4} \right]
\]  

(75)

\[
z_0^{\text{veg}} = \begin{cases} 
    z_0^{\text{wc}} + 0.3h_{\text{top}}^{\text{veg}} \left( c_d V_{ex} \right)^{1/2} & 0 \leq \left( c_d V_{ex} \right) \leq 0.2 \\
    0.3h_{\text{top}}^{\text{veg}} \left( 1 - \frac{d}{h_{\text{top}}^{\text{veg}}} \right) & 0.2 \leq \left( c_d V_{ex} \right) \leq 1.5
\end{cases}
\]  

(76)

where the \( c_d \) (\( \cdot \)) parameter defines the mean drag coefficient for individual leaves. We also implement the Raupach [1994] parameterization, not described here – see Raupach [1994] for details.

Given the above-canopy wind profile defined in equation (71), the aerodynamic resistance above the canopy can be calculated as
\[ r_a = \frac{1}{u_{air}C_H \phi_H^{ac}} \]  

(77)

where \( u_{air} \) (m s\(^{-1}\)) is the wind speed at the upper boundary, and \( C_H \) (-) is the exchange coefficient for heat, defined as

\[ C_H = \frac{k^2}{\ln \left( \frac{h_{ref} - d}{z_0}\right)^2} \]  

(78)

and \( \phi_H^{ac} \) (-) is the above-canopy stability correction as defined in equation (84). In the implementation here, the roughness length \( z_0 \) is assumed to be identical for momentum, heat, and moisture – in future work we will consider different parameterizations to reduce the roughness length for heat and moisture [Chen et al., 1997].

The below-canopy aerodynamic resistance can be defined for neutral stability using K-theory [Choudhury and Monteith, 1988; Niu and Yang, 2004; Flerchinger et al., 2012; Mahat et al., 2013] as

\[ r_{g,n} = \int_{h_{bot}}^{d + z_0} \frac{dh}{K_c} + \int_{z_0}^{b_{w}} \frac{dh}{K} \]  

(79)

where the height \( h = d + z_{0w} \) is the effective height of the canopy air space. In equation (79) \( K_c \) and \( K \) (m\(^2\) s\(^{-1}\)) define the eddy diffusivity within and below the canopy, and are given as

\[ K_c (h) = K_h (h_{top}) \exp \left[ -a_w \left( 1 - \frac{h}{h_{top}} \right) \right] \]  

(80)

\[ K (h) = \frac{k^2 u_{bot} h}{\ln \left( \frac{h_{bot}}{z_{0sfc}} \right)} \]  

(81)

Substituting equations (80) and (81) in equation (79), and integrating, provides [Mahat et al., 2013]

\[ r_{g,n} = \frac{h_{top} \exp(a_w)}{K_h (h_{top} w)} \left[ \exp \left( -a_w \frac{h_{top}}{h_{top}} \right) \exp \left( -a_w \frac{d+z_{0}}{h_{top}} \right) \right] + \frac{1}{k^2 u_{bot}^2} \ln \left( \frac{h_{bot}}{z_{0sfc}} \right)^2 \]  

(82)
where $K_s \left( h_{\text{top}} \right) = u^* \left( h_{\text{top}} - d \right)$ is the eddy diffusivity at the top of the canopy and $u^* = u_{\text{air}} \sqrt{C_H \phi_{H}^{bc}}$.

Stability corrections can then be applied as

$$r_g = \frac{r_{g,\Omega}}{\phi_{H}^{bc}}$$

(83)

where $\phi_{H}^{bc}$ is the below-canopy stability correction as defined in the next section.

The above- and below-canopy stability corrections $\phi_{H}^{\Omega}$ are defined as a continuous differentiable function of the bulk Richardson number $R_b^{\Omega}$ (-), i.e.,

$$\phi_{H}^{\Omega} = f \left( R_b^{\Omega} \right) \quad \Omega = ac, bc$$

(84)

where $ac$ and $bc$ define the above-canopy and the below-canopy stability corrections respectively. In SUMMA we use parameterizations from Anderson [1976], Louis [1979] and Mahrt [1987] to define the stability function in equation (84). Reba et al. [2014] conducted sensitivity analyses using alternative stability functions, and illustrated that the choice of stability function has a limited impact in the cases that they examined. Other stability functions based on Monin-Obukov theory can also be implemented and assessment of these functions is left for future work.

The Richardson number above and below the canopy is given as

$$R_b^{ac} = \frac{g \left( h_{\text{ref}} - d \right)}{\left( u_{\text{air}} - u_{zd} \right)^2} \frac{T_{\text{air}} - T_{\text{cas}}}{0.5\left( T_{\text{air}} + T_{\text{cas}} \right)}$$

(85)

$$R_b^{bc} = \frac{g h_{\text{bot}}^{\text{veg}}}{u_{\text{bot}}^{\text{veg}}} \frac{T_{\text{cas}} - T_{\text{surf}}}{0.5\left( T_{\text{cas}} + T_{\text{surf}} \right)}$$

(86)

where $h_{\text{ref}}$ and $h_{\text{bot}}^{\text{veg}}$ (m) represent, respectively, the height of the reference level (i.e., the height of the model forcing data) and the height at the bottom of the canopy, $u_{\text{air}}$, $u_{zd}$ and $u_{\text{bot}}^{\text{veg}}$ (m s$^{-1}$) are the wind speed at the reference level. The height of the zero plane displacement, and the bottom of the canopy, $T_{\text{air}}$, $T_{\text{cas}}$, and $T_{\text{surf}}$ (K) define, respectively, the temperature at the reference level, the canopy air space, and the surface, and $g$ (m s$^{-2}$) is the gravitational acceleration.
3.1.4.3 Leaf resistance

The *Choudhury and Monteith* [1988] parameterization of leaf resistance is used in many different models – see *Niu et al.* [2011] and *Mahat et al.* [2013] for recent examples. The conductance for individual leaves at a given height above the soil surface, \( g_l(h) \) (m s\(^{-1}\)) is

\[
g_l(h) = c_l \frac{u(h)}{w_l}
\]  

(87)

where \( c_l = 0.01 \) m s\(^{-1/2}\), \( u(h) \) is the wind speed height \( h \), and \( w_l \) (m) is the leaf width. Assuming leaf area is uniformly distributed throughout the canopy depth, the mean leaf conductance over the canopy \( \overline{g_l} \) (m s\(^{-1}\)) is

\[
\overline{g_l} = \frac{1}{h_{\text{top}}} \int_0^{h_{\text{top}}} g_l(h)dh
\]  

(88)

Given the exponential wind profile described by equation (71), and combining equations (87) and (88) and integrating, provides [*Choudhury and Monteith*, 1988; *Mahat et al.*, 2013]

\[
\overline{g_l} = \frac{2c_l}{a_w} \sqrt{\frac{u(h_{\text{top}})}{w_l}} \left[ 1 - \exp\left( -a_{w} \frac{a_w}{2} \right) \right]
\]  

(89)

where we recall that \( a_w \) is the exponential decay coefficient for wind speed through the canopy. The mean leaf resistance per unit ground area \( r_l \) (s m\(^{-1}\)) is then

\[
r_l = \frac{1}{\overline{g_l}}
\]  

(90)

where \( r_l \) is scaled by the exposed vegetation area index (leaf + stem) in equations (59) and (61) to provide the canopy conductance for heat and water.

3.1.4.4 Stomatal resistance and soil resistance

The stomatal resistance, \( r_{s,\text{sun}} \) and \( r_{s,\text{shd}} \) (s m\(^{-1}\)), and the soil resistance, \( r_{\text{soil}} \) (s m\(^{-1}\)), as used in equations (62) and (63), are described in Section 3.2.3.1. Briefly, water availability stress functions have a large limiting effect on both stomatal resistance and soil resistance – these functions are parameterized in different ways, and the choice of water availability stress functions can have a large impact on total
evapotranspiration (see also Niu et al. [2011]). Some parameterizations used here also include additional limiting factors in the stomatal resistance formulations beyond soil stress, including limitations of temperature, vapor pressure, and light [Jarvis, 1976; Ball et al., 1987].

### 3.1.5 Heat flux through snow and soil

We explicitly simulate the temperature throughout the snow and soil sub-domains, which is affected by fluxes at the upper boundary of the snow and soil domains (defined as the snow surface if snow is present, and the soil surface if there is no snow), the diffusive heat flux throughout the snow and soil domain, and the fluxes at the lower boundary. An important source of predictive differences in the diffusive heat flux is thermal conductivity $\lambda$ (W m$^{-1}$ K$^{-1}$). For snow, thermal conductivity is typically parameterized as an empirical function of snow density [e.g., Sturm et al., 1997]. In this work we consider the parameterizations from Yen [1965], Mellor [1977] and Jordan [1991], all of which have different functional representations of the relationship between thermal conductivity and snow density, and the parameterization of Smirnova et al. [2000] which specifies the thermal conductivity of snow as constant at $\lambda = 0.35$ W m$^{-1}$ K$^{-1}$. For soil, we parameterize thermal conductivity based on the volumetric fraction of different constituents $\lambda = \sum_k \theta_k \lambda_k$, $k = (\text{air}, \text{liq}, \text{ice}, \text{soil})$, where $\lambda_{\text{air}} = 0.026$, $\lambda_{\text{liq}} = 0.60$, $\lambda_{\text{ice}} = 2.5$, and $\lambda_{\text{soil}}$ is an adjustable parameter which depends on quartz content and other minerals. The formulation of the heat flux through snow and soil is presented in equation (3) (Section 2.1).

### 3.2 Hydrology

#### 3.2.1 Canopy Hydrology

##### 3.2.1.1 Liquid water drainage

Multiple methods exist to simulate the vertical flux of liquid water through the canopy. In this model we consider canopy drip as the only vertical liquid water flux, where $q_{\text{drip}}^{\text{veg}}$ (m s$^{-1}$) is defined as the rapid drainage of excess liquid water from the canopy. Following Bouten [1996]

$$q_{\text{drip}}^{\text{veg}} = \begin{cases} \kappa_{\text{drip}}^{\text{veg}} \frac{W_{\text{liq}}^{\text{veg}} - I_{b,\text{liq}} V_{ex}}{\rho_{\text{liq}}} & W_{\text{liq}}^{\text{veg}} > I_{b,\text{liq}} V_{ex} \\ 0 & W_{\text{liq}}^{\text{veg}} \leq I_{b,\text{liq}} V_{ex} \end{cases} \quad (91)$$

where $\kappa_{\text{drip}}^{\text{veg}}$ (s$^{-1}$) is the time constant for canopy drip, $V_{ex}$ (m$^2$ m$^{-2}$) is the exposed vegetation area index (stem plus leaf), i.e., the vegetation surface that is still exposed after accounting for snow accumulation.
below the vegetation canopy, and \( I_{b,liq} \) (kg m\(^{-2}\)) is the branch interception capacity of liquid water per unit \( V_{ex} \). In equation (91) \( I_{b,liq} \) is an empirical parameter – Niu and Yang [2004] suggest using a default value of \( I_{b,liq} = 0.1 \) kg m\(^{-2}\).

### 3.2.1.2 Throughfall of snow

Throughfall of snow through the canopy is typically expressed as a linear function of maximum interception capacity [Hedstrom and Pomeroy, 1998; Andreadis et al., 2009]

\[
q_{\text{veg fall}} = q_{sf} \frac{W_{icke}^{\text{veg}}}{I_{b,ice} V_{ex}}
\]

where \( I_{b,ice} \) is interception capacity of snow per unit \( V_{ex} \) and parameterized as:

\[
I_{b,ice} = \begin{cases} 
T_{b,ice} \left( 0.27 + \frac{46}{\rho_{s,new}} \right) & \text{Hedstrom and Pomeroy (1988)} \\
ml_r & \text{Andreadis et al. (2009)} 
\end{cases}
\]

where \( T_{b,ice} \) (kg m\(^{-2}\)) is the reference interception capacity for snow, \( m \) (kg m\(^{-2}\)) is an empirical parameter determined based on observations of maximum interception capacity, \( \rho_{s,new} \) (kg m\(^{-3}\)) is the density of new snow, formulated as an empirical function of temperature [Hedstrom and Pomeroy, 1998]

\[
\rho_{s,new} = 67.92 + 51.25 \exp \left( \frac{T_{air} - T_{frz}}{2.59} \right)
\]

where \( T_{frz} = 273.16 \) K defines the freezing point of pure water.

\( L_r \) (-) describes the impact of temperature on interception efficiency [Andreadis et al., 2009]

\[
L_r = \begin{cases} 
4.0 & (T_{air} - T_{frz}) > -1 \\
1.5(T_{air} - T_{frz}) + 5.5 & -1 \leq (T_{air} - T_{frz}) < -3 \\
1.0 & (T_{air} - T_{frz}) \leq -3 
\end{cases}
\]

Mahat and Tarboton [2013] use values of \( T_{b,ice} \) of 6.6 and 5.9 kg m\(^{-2}\) for pine and spruce trees respectively, which, making use of equation (93), provides values of \( I_{b,ice} \) ranging from \(~3.5-6.2\) kg m\(^{-2}\).
(pine) and ~3.1-5.5 kg m\(^{-2}\) (spruce). Andreadis et al. [2009] use a default value of \(m = 5\) kg m\(^{-2}\), which, again making use of equation (93), provides values of \(I_{b,\text{ice}}\) ranging from 5-20 kg m\(^{-2}\).

### 3.2.1.3 Unloading of snow

Unloading of snow from the forest canopy is also parameterized differently across models:

\[
q_{\text{u,\text{linear}}}^{\text{veg}} = \kappa_{\text{unload}}^{\text{veg}} W_{\text{ice}}^{\text{veg}}
\]

\[
q_{\text{u,drip}}^{\text{veg}} = \begin{cases} 
0 & W_{\text{ice}}^{\text{veg}} \leq W_{\text{ice,\text{res}}}^{\text{veg}} \\
\chi_u q_{\text{drip}}^{\text{veg}} & W_{\text{ice}}^{\text{veg}} > W_{\text{ice,\text{res}}}^{\text{veg}}
\end{cases}
\]

where equation (96) is the parameterization used by Hedstrom and Pomeroy [1998] and Mahat and Tarboton [2013], and equation (97) is the parameterization used by Andreadis et al. [2009], where \(q_{\text{drip}}^{\text{veg}}\) is defined in equation (91). Here \(\kappa_{\text{unload}}^{\text{veg}}\) (s\(^{-1}\)) is the time constant for unloading, \(\chi_u\) (\(\cdot\)) is an empirical parameter that relates the ratio of mass release to meltwater drip, and \(W_{\text{ice,\text{res}}}^{\text{veg}}\) (kg m\(^{-2}\)) is the residual intercepted snow that can only be melted or sublimated off the canopy. Andreadis et al. [2009] suggest \(\chi_u = 0.4\) and \(W_{\text{ice,\text{res}}}^{\text{veg}} = 5\) kg m\(^{-2}\) based on observations in Oregon, USA. Note in equation (91) that meltwater drip (and hence unloading) only occurs after the liquid water holding capacity of vegetation is satisfied.

Since unloading of snow in the absence of canopy melt and unloading of snow during canopy melt events may both occur at a single site, we combine these different options into a single parameterization

\[
q_{\text{unload}}^{\text{veg}} = q_{\text{u,drip}}^{\text{veg}} + q_{\text{u,\text{linear}}}^{\text{veg}}
\]

where the Hedstrom and Pomeroy [1998] option can be obtained by setting \(\chi_u = 0\), and the Andreadis et al. [2009] option can be obtained by setting \(\kappa_{\text{unload}}^{\text{veg}} = 0\).

### 3.2.2 Snow hydrology

Following Colbeck [1976] and Colbeck and Anderson [1982], the vertical flux of liquid water within the snowpack is parameterized as gravity drainage,

\[
q_{\text{liq},z}^{\text{snow}} = K^{\text{snow}} \left( \theta_{\text{liq}}^{\text{snow}} \right)
\]
where $K^{\text{snow}}$ is the hydraulic conductivity of the snow. This expression for liquid water flux neglects the capillary term based on the premise that gravity forces dominate capillary forces.

Many different functions can be used to relate $K^{\text{snow}}$ to $\theta^{\text{snow}}$ [Dall’Amico et al., 2011]. In this paper the hydraulic conductivity of snow is defined using the Brooks and Corey [1964] relation

$$K^{\text{snow}}(\theta^{\text{snow}}) = K_{\text{sat}}^{\text{snow}} \left( \frac{\theta^{\text{snow}}}{\theta_{\text{sat}}^{\text{snow}}} - \frac{\theta_{\text{res}}^{\text{snow}}}{\theta_{\text{sat}}^{\text{snow}}} \right)^C$$

(100)

where $K_{\text{sat}}^{\text{snow}}$ (m s$^{-1}$) is the saturated hydraulic conductivity of snow, $\theta_{\text{sat}}^{\text{snow}}$ (-) is the porosity of snow, $\theta_{\text{res}}^{\text{snow}}$ (-) is the irreducible liquid water in the snowpack, and $C$ (-) is an exponent related to the pore size distribution. In equation (100) $\theta_{\text{res}}^{\text{snow}} = \phi_{\text{res}} \theta_{\text{sat}}^{\text{snow}}$, where the parameter $\phi_{\text{res}}$ defines the fraction of pore space that must filled before drainage of liquid water can occur.

Here the liquid water flow is solved using the finite difference approximation described later. Other solution methods are of course possible – for example, the kinematic method of Dunne et al. [1976] – which can have impacts on the coupling between thermodynamics and hydrology. The impact of other numerical solution schemes is deferred to a further study.

### 3.2.3 Soil hydrology

#### 3.2.3.1 Transpiration

Modeled transpiration losses from the soil depend on multiple modeling decisions, especially the parameterizations of within-canopy eddy diffusivity and the wetted area of the canopy (as described in Section 3.1.4). As given in equations (55) and (62), the rate of transpiration is limited by the stomatal resistances $r_{s,\text{sun}}$ and $r_{s,\text{shd}}$. The next sections consider water availability controls on stomatal resistance (this is the only control defined by Liang et al. [1994]), and additional physiological controls on stomatal resistance, as used in models including CLM and Noah-MP.

#### 3.2.3.1.1 Water availability controls on stomatal resistance

The parameterization of stomatal resistance differs markedly across models. The simplest approach considered here is similar to that used in the VIC model [Liang et al., 1994], where stomatal resistance is parameterized solely as a function of soil moisture limitations, and stomatal resistance is identical for sunlit and shaded leaves. The simple resistance parameterization can be given as
\[ r_{s,sun} = r_{s,shd} = \frac{r_{oc}}{\bar{\beta}_v} \tag{101} \]

where \( r_{oc} \) (s m\(^{-1}\)) is the minimum stomatal resistance, and \( \bar{\beta}_v \) (-) is the total soil water stress function, defined as

\[
\bar{\beta}_v = \sum_j f_{roots,j} \beta_{v,j} + f_{roots}^{aq} \beta_v^{aq} \tag{102}
\]

where \( z_{soil} \) is the soil depth, and \( f_{roots,j} \) and \( \beta_{v,j} \) are, respectively, the root density and the water availability stress function in the \( j \)-th soil layer, and \( f_{roots}^{aq} \) and \( \beta_v^{aq} \) are respectively the fraction of roots and water availability stress function for the aquifer.

Equation (102) allows for some fraction of roots below the resolved depth of the soil profile, i.e.,

\[
\sum_j f_{roots,j} \leq 1 \tag{103}
\]

meaning that the fraction of roots in the aquifer can be given as

\[
f_{roots}^{aq} = 1 - \sum_j f_{roots,j} \tag{104}
\]

where the root density at a given depth is parameterized to decrease up to a prescribed rooting depth, \( z_{root} \) (m). The total fraction of roots above depth \( z_x \) is defined as

\[
\frac{1}{z_x} \int_0^{z_x} f_{roots}(z) dz = \left( \frac{z_x}{z_{root}} \right)^{\omega_{root}} \quad z_x \leq z_{root} \tag{105}
\]

where \( \omega_{root} \) defines the shape of the rooting profile.

An important term in equation (102) is the water availability stress function, which varies markedly across models [e.g., see Mahfouf et al., 1996]. Here we implement the three parameterizations used in Noah-MP [Niu et al., 2011].
\[
\beta_i(z) = \begin{cases} 
\frac{\theta_{lq}(z) - \theta_{lq,\text{wilt}}}{\theta_{lq,\text{ref}} - \theta_{lq,\text{wilt}}} & \text{Noah - type} \\
\frac{\psi_{\text{wilt}} - \psi(z)}{\psi_{\text{wilt}} - \psi_{\text{sat}}} & \text{CLM - type} \\
1 - \exp \left[ -c_2 \ln \left( \frac{\psi_{\text{wilt}}}{\psi(z)} \right) \right] & \text{SSiB - type}
\end{cases}
\] (106)

where \( \beta_i(z) \) is constrained to be between zero and one. The options in equation (106) come from Noah [Chen et al., 1997], CLM [Oleson et al., 2010], and the Simplified Simple Biosphere (SSiB) model [Xue et al., 1991]. In equation (106) \( \theta_{lq}(z) \) (-) and \( \psi(z) \) (m) are the volumetric liquid water content and matric head at depth \( z \), \( \theta_{lq,\text{wilt}} \) (-) and \( \psi_{\text{wilt}} \) (m) are the volumetric liquid water content and matric head at the wilting point (which can depend on vegetation type), \( \theta_{lq,\text{ref}} \) (-) is a reference volumetric liquid water content value above which there is no water stress on vegetation, \( \psi_{\text{sat}} \) (m) is the matric head at saturation (\( \psi_{\text{sat}} = 0 \) when using the van Genuchten functions, as used here). The \( c_2 \) parameter in the SSiB-type model ranges from approximately 4-7, depending on vegetation type [Xue et al., 1991; Niu et al., 2011].

3.2.3.1.2 Physiological stomatal resistance parameterizations

More complex parameterizations of stomatal resistance have more complete representations of the physiological factors controlling transpiration. Here we consider the parameterizations of stomatal resistance included in the Noah-MP model [Niu et al., 2011] – these include the Jarvis [1976] parameterization as used in the Noah land-surface model [Sellers, 1985; Noilhan and Planton, 1989; Chen et al., 1996], and the Ball et al. [1987] parameterization as used in CLM [Bonan, 1996; Oleson et al., 2010].

The Jarvis parameterization is [Jarvis, 1976; Sellers, 1985; Noilhan and Planton, 1989; Chen et al., 1996]

\[
r_{s,i} = \frac{r_{sc}}{f(Q_{\text{PAR},i})f(T_{\text{air}})f(e_d)\bar{B}_v} \quad i = \text{sun,shd}
\] (107)

where the subscript \( i \) defines either sunlit or shaded leaves. The functions \( f(Q_{\text{PAR},i}) \), \( f(T_{\text{air}}) \), and \( f(e_d) \), all limited to the range 0-1, represent the effects of photosynthetically-active radiation (PAR), air
temperature, and vapor pressure deficit, where $Q_{PAR,j}$ represents PAR absorbed on sunlit or shaded leaves, as defined in equations (65) and (66).

The Ball-Berry parameterization is described in detail by Oleson et al. [2010] and Niu et al. [2011], so only brief details are provided here. Stomatal conductance per unit sunlit and shaded leaf area, $g_i$ ($\mu$mol m$^{-2}$ s$^{-1}$) is a function of the rate of photosynthesis $A_i$ ($\mu$mol m$^{-2}$ s$^{-1}$), given as

$$g_i = \nu_i \frac{A_i}{c_{air} \cdot e_{sat}(T_{veg})} P_{air} + g_{min} \bar{P}_v \quad i = \text{sun, shd} \quad (108)$$

where $c_{air}$ (Pa) is the CO$_2$ concentration at the leaf surface (time varying model forcing, representing carbon fertilization), $g_{min}$ ($\mu$mol m$^{-2}$ s$^{-1}$) is the minimum stomatal conductance, and $\nu_i$ (-) is an empirical parameter to relate transpiration to the CO$_2$ flux, where a greater value of $\nu_i$ means the leaf consumes more water to produce the same carbon mass [Niu et al., 2011].

The rate of photosynthesis, $A_i$ ($\mu$mol m$^{-2}$ s$^{-1}$), is given as [Oleson et al., 2010; Niu et al., 2011]

$$A_i = I_{gs} \min(A_c, A_{L,i}, A_s) \quad i = \text{sun, shd} \quad (109)$$

where $I_{gs}$ is a growing season index ($I_{gs} = 1$ if the vegetation temperature is above the minimum temperature for transpiration, and $I_{gs} = 0$ otherwise), and $A_c$, $A_{L,i}$, and $A_s$ are carboxylase-limited, light-limited, and export-limited photosynthesis rates, where the controls of soil moisture and vegetation temperature are included in the parameterizations of $A_c$ and $A_s$ [see Oleson et al., 2010 for complete details].

3.2.3.1.3 Transpiration from soil layers

Transpiration is included in the soil hydrology calculations through use of a sink term obtained as part of the spatial discretization described in Clark et al. [2015b]. The transpiration sink term $(S_{et}^{soil})_j$ (s$^{-1}$) is computed for a given soil layer $j$ as

$$\left(S_{et}^{soil}\right)_j = \left(f_{roo}\right)_j \left(\beta_v\right)_j \frac{Q_{trans}^{veg}}{L_{cap} \rho_{liq} (\Delta z)_j} + \left(S_{evap}^{soil}\right)_j \quad (110)$$
where $Q_{\text{trans}}$ (W m$^{-2}$) is the transpiration flux, $(\beta_j)$ is the soil water stress for the $j$-th soil layer as defined in equation (106), $\bar{\beta}$ is the total water availability stress factor as defined in equation (102), including stress from both the soil and the aquifer, $(f_{\text{root}})_j$ is the fraction of roots in the $j$-th soil layer defined in equation (105), $(\Delta z)_j$ is the depth of the $j$-th soil layer, and $L_{\text{wet}}$ (J kg$^{-1}$) and $\rho_{\text{liq}}$ (kg m$^{-3}$) are respectively the latent heat of vaporization and the intrinsic density of liquid water. The second term in equation (110), $(S_{\text{evap}})_j$ (s$^{-1}$) is the ground evaporation (only defined for the upper-most soil layer) and is defined in the next section.

### 3.2.3.2 Ground evaporation

The ground evaporation flux, $Q_{l\text{f}}$ (W m$^{-2}$), is defined in equation (56) in Section 3.1.4, which depends on the ground aerodynamic resistance defined in equation (63), as well as the soil resistance and relative humidity in the soil pore space. When snow is present, soil evaporation is set to zero, and evaporation from the snowpack is computed using $r_{\text{soil}} = 0$ and $\phi_{\text{hum}} = 1$.

Many models use the parameterizations of soil resistance and soil humidity from CLASS and SSiB [Sellers et al., 1986; Verseghy, 1991]. Here we follow this widely used approach, and only provide a single option for soil resistance and soil humidity to compute $Q_{l\text{f}}$, given as

$$r_{\text{soil}} = (1 - f_{\text{snow}}) \exp \left( x_{r0} - x_{r1} \frac{\theta_{\text{soil}(1)}^{\text{sat}}}{\theta_{\text{soil}(1)}^{\text{sat}} - \theta_{\text{soil}(1)}^{\text{res}}} \right) \quad (111)$$

$$\phi_{\text{hum}} = f_{\text{snow}} + (1 - f_{\text{snow}}) \exp \left( \frac{g \psi_{\text{soil}(1)}}{R_m T_{\text{soil}(1)}} \right) \quad (112)$$

where $f_{\text{snow}}$ is the fractional snow covered area (in the current implementation $f_{\text{snow}}$ is either zero or one, denoting presence/absence of snow, and a non-binary representation is also possible [Clark et al., 2011]), $\theta_{\text{soil}}^{\text{sat}}$ and $\theta_{\text{soil}}^{\text{res}}$ (-) define soil porosity and residual volumetric liquid water content, $\theta_{\text{liq}}^{\text{soil}(1)}$ (-) and $\psi_{\text{soil}(1)}$ (m) are the volumetric liquid water content and the matric head for the upper-most soil layer, $T_{\text{soil}(1)}$ (K) is the temperature of the upper-most soil layer, and the constants $g$ (m s$^{-2}$) and $R_m$ (J kg$^{-1}$ K$^{-1}$) are the gravitational acceleration and the gas constant for water vapor. The default values for the empirical parameters in equation (111) are $x_{r0} = 8.25$ and $x_{r1} = 4.225$, following Sellers et al. [1992].
The ground evaporation term in equation (110), \( \left( S_{\text{evap}}^{\text{soil}} \right)_j \) \( (s^{-1}) \) can then be computed as

\[
\left( S_{\text{evap}}^{\text{soil}} \right)_j = \frac{Q_{\text{fc}}^S}{L_{\text{ap}} P_{\text{liq}} (\Delta z)_j} \quad j = 1, n_{\text{snow}} = 0
\]  

(113)

where ground evaporation is only extracted from the upper-most soil layer when the ground surface is free of snow \( (n_{\text{snow}} \) is the number of snow layers).

### 3.2.3.3 Infiltration

Infiltration of water into soil depends on multiple modeling decisions \cite{Clark et al., 2008}, especially the point infiltration rate, the fractional area of the model element that is saturated, and the fraction of the model element that is impermeable due to partially frozen soils. In this work we use the Green-Ampt approach for the point infiltration rate \cite{Green and Ampt, 1911}, a parameterization similar to Wood et al. \cite{Wood et al., 1992} to estimate the saturated fraction, and the parameterization of Koren et al. \cite{Koren et al., 1999} to estimate the impermeable fraction associated with frozen soils.

The total influx to the soil can be written as

\[
q_{\text{infl}}^{\text{soil}} = (1 - A_f) (1 - A_s) q_{\text{infl},p}^{\text{soil}}
\]

(114)

where \( A_f \) \(-\) defines the fractional impermeable area associated with soil freezing, \( A_s \) \(-\) is the fractional saturated area over the non-frozen part of the grid cell or basin, and \( q_{\text{infl},p}^{\text{soil}} \) \((m \cdot s^{-1}) \) is the point infiltration rate. Different models have different parameterizations of \( A_f \), \( A_s \), and \( q_{\text{infl},p}^{\text{soil}} \) \( (\text{e.g., see Clark et al., 2008}) \), for different parameterizations of \( A_s \), and for considerations of expedience, we only provide a single option for infiltration, and plan to extend and evaluate alternative infiltration parameterizations in future work.

The fractional impermeable area associated with soil freezing is computed following the approach of Koren et al. \cite{Koren et al., 1999}. Given an estimate of the spatial mean of the fraction of ice in the root zone, i.e.,

\[
\overline{W}_{\text{ice}}^{\text{soil}} = \int_0^{z_{qs}} \rho_{\text{ice}}^{\text{soil}} dz
\]

(115)

where the upper limit of the integral in equation (115), \( z_{qs} \) \((m)\), is the depth of soil considered for the surface runoff computations (typically \( z_{qs} \) is set to the depth of the root zone). Assuming the spatial
variability of freezing depth follows a Gamma distribution, the fractional impermeable area associated with soil freezing, $A_f$ (-) is

$$A_f = \frac{1}{\Gamma(a)} \int_0^v e^{-x} x^{a-1} dx$$  \hspace{1cm} (116)$$

where the upper limit of the integral in equation (116) is

$$v = a \frac{W_{soil, cr}}{W_{ice}}$$ \hspace{1cm} (117)$$

with $a = 1/C_v^2$ and $W_{soil, cr}$ (m) is the critical soil ice content. Equation (116) is solved using the incomplete Gamma function [Press et al., 1992].

The fractional saturated area over the non-frozen part of the grid cell or basin, $A_s$ (-), is calculated considering the water content over the soil depth $z_{qs}$ (typically $z_{qs}$ is set to the depth of the root zone), as

$$A_s = \exp\left(-\chi_{qscal} \phi_{qs}\right)$$ \hspace{1cm} (118)$$

where $\chi_{qscal}$ (-) is a scaling factor, and $\phi_{qs}$ (-) is the unfrozen pore space over the soil depth $z_{qs}$ that is filled with liquid water, given as

$$\phi_{qs} = \frac{\bar{W}_{soil}}{\theta_{soil} z_{qs} - \bar{W}_{ice}}$$ \hspace{1cm} (119)$$

where, similar to equation (115),

$$\bar{W}_{soil} = \int_0^{z_{qs}} \theta_{soil} dz$$ \hspace{1cm} (120)$$

Equation (118) is similar to the VIC parameterization [Liang et al., 1994], except we select the exponential function rather than the power function in VIC for computational simplicity.

The point infiltration rate, $q_{infl,p}$ (m s$^{-1}$) is computed using a simplified Green-Ampt approach [Green and Ampt, 1911]. The maximum infiltration rate $q_{infl,max}$ (m s$^{-1}$) is given as

$$q_{infl,max} = k_{z_{wet}} \left( \frac{\psi_{ga} + z_{wet}}{z_{wet}} \right)$$ \hspace{1cm} (121)$$
where \( k_{\text{wet}} \) (m s\(^{-1}\)) is the hydraulic conductivity at the depth of the wetting front, \( \psi_{\text{ga}} \) (m) is the Green-Ampt wetting front suction, and \( z_{\text{wet}} \) (m) is the depth to the wetting front. The depth to the wetting front is parameterized simply as

\[
z_{\text{wet}} = \phi_{qs} z_{qs}
\]

where \( \phi_{qs} \) is computed from equation (119). Recall that \( z_{qs} \) is the depth of soil considered for the surface runoff computations.

The point infiltration rate is then given as

\[
q_{\text{infl,p}}^{\text{soil}} = \min\left(q_{\text{rm}}, q_{\text{infl,max}}^{\text{soil}}\right)
\]

where \( q_{\text{rm}} \) (m s\(^{-1}\)) is the sum of rainfall, \( q_{nf} \), and drainage from the base of the snowpack.

### 3.2.3.4 Vertical redistribution

Our spatial approximation of the 3D flow equations requires the use of different approaches to simulate the vertical and lateral fluxes of water through soil. The vertical redistribution of water among soil layers (in a single soil column) is based on the mixed form of Richards equation [Celia et al., 1990], using the van Genuchten constitutive functions for soil water retention and hydraulic conductivity [Van Genuchten, 1980], with the option for saturated hydraulic conductivity to decrease with depth according to power-law transmissivity profiles [Duan and Miller, 1997; Iorgulescu and Musy, 1997]. Other methods can be incorporated within this framework and will be explored in future work.

We currently use the Darcy flux as the single model option for vertical redistribution of liquid water, using infiltration computed using the Green-Ampt approach as the boundary condition at the top of the soil column. The Darcy flux is given as

\[
\left(q_{\text{liq}}^{\text{soil}}\right)^{\text{soil}} = -K^{\text{soil}} \frac{\partial \psi^{\text{soil}}}{\partial z} + K^{\text{soil}}
\]

where the two components of the liquid water flux are the capillary and gravity fluxes, \( \psi^{\text{soil}} \) (m) is the matric head, and \( K^{\text{soil}} \) (m s\(^{-1}\)) is the unsaturated hydraulic conductivity of soil.

#### 3.2.3.4.1 Constitutive functions

Combining equation (124) with equation (8), and ignoring flow in the \( x \) and \( y \) dimensions, provides the one-dimensional Richards equation, for which a solution requires constitutive functions to relate \( \Theta_{\text{liq}}^{\text{soil}} \) to
\( \psi_{\text{soil}} \), and \( K_{\text{soil}} \) to \( \psi_{\text{soil}} \). For this initial implementation we use the van Genuchten [1980] functions, defined for unfrozen soils as (the superscript soil is omitted here for generality)

\[
\theta_{\text{liq}}(\psi) = \frac{\theta_{\text{sat}} - \theta_{\text{res}}}{1 + (\alpha_{vg} \psi)^{n_{vg}}} + \theta_{\text{res}}
\]

(125)

\[
K_{\text{unf}}(\psi, z) = K_{\text{sat}}(z) \left\{ \frac{1 - (\alpha_{vg} \psi)^{n_{vg}}}{1 + (\alpha_{vg} \psi)^{n_{vg}}} \right\}^{2}
\]

(126)

where \( \theta_{\text{sat}} \) and \( \theta_{\text{res}} \) define the porosity and residual volumetric liquid water content, \( \alpha_{vg} \) (m\(^{-1}\)) defines the capillary length scale, and the parameters \( m_{vg} \) (-) and \( n_{vg} \) (-) are related to the pore size distribution \( m_{vg} = 1 - 1/n_{vg} \), and \( K_{\text{sat}}(z) \) (m s\(^{-1}\)) is the saturated hydraulic conductivity of soil at depth \( z \). In equation (125), \( \theta_{\text{liq}}(\psi) = \theta_{\text{sat}} \) for \( \psi \geq 0 \).

Transmission for partially frozen soil is modified from the van Genuchten form (or any other form) in many different ways [Zhang et al., 2010]. Here we do so by including an ice impedance factor, \( f_{\text{ice}} \) (-), which reduces the hydraulic conductivity when ice is present [e.g., Zhang et al., 2007], as

\[
K(\psi, \theta_{\text{soil}}) = K_{\text{unf}}(\psi) \times 10^{-f_{\text{ice}} \psi_{\text{soil}}}
\]

(127)

where \( K_{\text{unf}} \) (m s\(^{-1}\)) is the unfrozen hydraulic conductivity of soil as defined in equation (124).

### 3.2.3.4.2 Decrease in hydraulic conductivity with depth

A key modeling decision is how to represent changing hydraulic conductivity with depth. Many traditional land-surface models compute hydraulic conductivity based only on soil texture [e.g., Oleson et al., 2010], whereas many hydrologic models impose a decrease of saturated hydraulic conductivity with increasing soil depth [e.g., Beven, 1997]. Given a power-law transmissivity profile [e.g., Duan and Miller, 1997], the saturated hydraulic conductivity \( K_{\text{sat}}(z) \) (m s\(^{-1}\)) at a given soil depth \( z \) (m) is given by Rupp and Woods [2008] as

\[
K_{\text{sat}}(z) = K_{\text{sat}}^0 \left( 1 - \frac{z}{z_{\text{soil}}} \right)^{n_{sf}^{-1}} \quad n_{sf} \geq 1
\]

(128)
where $K_{sat}^0$ (m s$^{-1}$) is the saturated hydraulic conductivity at the soil surface, $z_{soil}$ (m) is the depth of the soil profile, and $n_{sf}$ (-) is the key model parameter that describes the shape of the transmissivity profile. Note from equation (128) that $K_{sat}^{soil}$ is constant with depth for $n_{sf} = 1$, i.e., $K_{sat}^{soil}(z) = K_{sat}^0$ for all soil depths, and $K_{sat}^{soil} = 0$ at the base of the soil profile for $n_{sf} > 1$.

One possible modeling option is to include a conceptual subterranean aquifer at the base of the soil profile. This option is currently only implemented for the case $n_{sf} = 1$, because hydraulic conductivity at the bottom of the soil profile is zero when $n_{sf} > 1$.

### 3.2.3.5 Lateral flux from the soil profile

The soil columns can be hydrologically connected, such that the lateral flux from upslope soil columns is the inflow to downslope soil columns, or hydrologically-disconnected (using one or many soil columns), in which case the lateral flux of water from soil columns is assumed to flow directly into the river network.

#### 3.2.3.5.1 Background

The continuity equation for sub-surface storage (i.e., below the water table) can be written for a given model element as [Wigmosta et al., 1994]

$$\phi_{dr} \frac{dz_{wt}}{dt} = \frac{Q_{out} - Q_{in}}{A} - q_{rchg} \tag{129}$$

where $\phi_{dr} = (\theta_{sat}^{soil} - \theta_{fc}^{soil})$ is the “drainable” porosity, $\theta_{fc}^{soil}$ is the field capacity of soil, $z_{wt}$ (m) is the depth to the water table, and $Q_{in}$ and $Q_{out}$ (m$^3$ s$^{-1}$) are the lateral inflow and outflow, $q_{rchg}$ (m$^3$ s$^{-1}$) is the vertical recharge rate, and $A$ (m$^2$) is the element area.

Assuming an unconfined horizontal aquifer, the rate of saturated sub-surface flow from the soil profile $Q_{out}$ (m$^3$ s$^{-1}$) is given by Wigmosta and Lettenmaier [1999] as

$$Q_{out} = x_{len} \tan(\beta) T(z_{wt}) \tag{130}$$

where $x_{len}$ (m) is the flow width for a given model element, $\beta$ (-) is the water table slope, and $T(z_{wt})$ (m$^2$ s$^{-1}$) is the aquifer transmissivity at water table depth $z_{wt}$. A power-law transmissivity profile is
obtained by integrating the saturated hydraulic conductivity profile \( K_{\text{sat}}(z) \) given by equation (128) from the water table depth \( z_{\text{wt}} \) to the bottom of the soil profile \( z_{\text{soil}} \) (noting that \( z_{\text{wt}} \leq z_{\text{soil}} \)) as

\[
T(z_{\text{wt}}) = \int_{z_{\text{wt}}}^{z_{\text{soil}}} K_{\text{sat}}(w) dw
\]

where \( w \) is a dummy variable of integration. This yields the transmissivity function

\[
T(z_{\text{wt}}) = \frac{K_0^\text{soil} z_{\text{soil}}}{n_{sf}} \left( 1 - \frac{z_{\text{wt}}}{z_{\text{soil}}} \right)^{n_{sf}} \quad n_{sf} > 1
\]

as used in some applications of TOPMODEL [Duan and Miller, 1997]. In equations (128) through (132) \( K_0^\text{sat} \) (m s\(^{-1}\)) is the saturated hydraulic conductivity at the soil surface, \( z_{\text{soil}} \) (m) is the depth of the soil profile, and \( n_{sf} \) (-) is the key model parameter that describes the shape of the transmissivity profile. Note from equation (128) that \( K_{\text{sat}}^\text{soil} \) is constant with depth for \( n_{sf} = 1 \), i.e., \( K_{\text{sat}}^\text{soil}(z) = K_0^\text{sat} \) for all soil depths, and \( K_{\text{sat}}^\text{soil} = 0 \) at the base of the soil profile for \( n_{sf} > 1 \).

### 3.2.3.5.2 Storage-based implementation to represent lateral flow between soil columns

In contrast to Wigmosta et al. [1994], the approach considered here formulates the subsurface flow equations in terms of volumetric liquid water content within the soil profile, without ever calculating the vertical position of the water table. This is done in order to simplify the calculations, since the water table and soil moisture share the same physical space. Equation (129), therefore, is never implemented directly in this current work, and is only provided as background for the derivations of the storage-based approach implemented here.

Consistent with the definition of drainable porosity in equation (129), the “drainable” water storage and the maximum drainable water storage can be given as

\[
W_{\text{dr}}^\text{soil} = \int_{z_{\text{wt}}}^{z_{\text{soil}}} \left[ \theta_{\text{liq}}(z) - \theta_{\text{fc}}^\text{soil} \right] dz
\]

\[
W_{\text{dr,max}}^\text{soil} = \phi_{\text{dr}} z_{\text{soil}}
\]
where $\theta_{\text{liq}}(z)$ is the volumetric liquid water content at soil depth $z$, and the lower limit of the integral in equation (133), $z_{\text{crit}}$, is the lowest point in the soil profile where $\theta_{\text{liq}} < \theta_{\text{fc}}$ (meaning that $\theta_{\text{liq}} > \theta_{\text{fc}}$ from $z_{\text{crit}}$ to the bottom of the soil profile).

The total lateral outflow, $Q_{\text{out}}$ (m$^3$ s$^{-1}$), can then be given as

$$Q_{\text{out}} = x_{\text{tan}} \tan(\beta) \frac{K_{\text{sat}}^0 \frac{W_{\text{soil}}}{W_{d,\text{max}}}^{n_{sf}}}{\phi d r n_{sf}}$$

(135)

where $\beta$ is the gradient in the land surface, used to approximate the water table gradient (the kinematic approximation). This provides the total lateral flux

$$q_{\text{base}}^{\text{soil}} = \frac{Q_{\text{out}} - Q_{\text{in}}}{A}$$

(136)

The total lateral flux $q_{\text{base}}^{\text{soil}}$ can then be apportioned to individual soil layers, obtained after spatial discretization described in Clark et al. [2015b], to provide the lateral flow sink term

$$\left(S_{y}\right)_j = \left(w_r\right)_j q_{\text{base}}^{\text{soil}}$$

(137)

where $\left(w_r\right)_j$ is the ratio of the transmissivity of the $j$-th layer to the total transmissivity.

3.2.3.5.3 Hydrologic connectivity

As stated above, the lateral fluxes from the soil columns can be hydrologically connected, in which the lateral flux from upslope soil columns is the inflow to downslope soil columns, or hydrologically disconnected (using one or many soil columns), in which the lateral flux of water from soil columns is assumed to flow into the river network. These modeling approaches are obtained as follows:

1. The hydrologically connected implementation can be obtained as a sequence of 1D solutions because of the kinematic approximation in equation (135) – the solution is obtained by first computing the 1D solution for each hillslope element (or upslope grid cell) before computing the solution for the riparian element (or downslope grid cell); and

2. The hydrologically-disconnected implementation is obtained assuming there is no lateral inflow into a soil column. The lateral outflux from the soil column can be given as
where $\chi_L$ (m$^{-1}$) is a length scale. This spatially disconnected case is similar to the TOPMODEL implementation of Niu et al. [2005], except we do not include an aquifer below the bottom of the soil profile because the power law transmissivity profile in equation (128) provides $K_{\text{sat}} = 0$ at the base of the soil profile.

Both the hydrologically-connected and the hydrologically-disconnected implementations use the power-law decrease in saturated hydraulic conductivity described by equation (128) to describe vertical fluxes of liquid water within the soil profile, with $n_{sf} > 1$.

### 3.2.4 Lateral flux from the subterranean aquifer

Another available modeling option is a conceptual subterranean aquifer at the base of the soil profile. This option is only included for the linear transmissivity profile, that is, the case of $n_{sf} = 1$ in equation (128), because the power-law transmissivity profile ($n_{sf} > 1$) leads to zero flux lower boundary condition at the base of the soil profile.

The aquifer recharge flux assumes a unit head gradient across the soil column-aquifer interface and is represented as

$$q_{\text{rech}}^a = \phi_b K_{\text{soil}}^a (z_{\text{soil}})$$

(139)

where $K_{\text{soil}}^a (z_{\text{soil}})$ is the unsaturated hydraulic conductivity at the base of the soil profile, and $\phi_b$ (-) is an anisotropic factor typically related to topographic slope, where $\phi_b < 1$ constricts drainage at the bottom of the soil profile. Note that equation (139) is used as the bottom boundary condition in the soil profile (in the VIC model $\phi_b = 1$, which defines a free drainage lower boundary condition).

The baseflow from the aquifer to the stream is represented as [Clark et al., 2008; Clark et al., 2009]

$$q_{\text{base}}^a = K_{\text{sat}}^a \left( \frac{S_{\text{crit}}^a}{S_{\text{crit}}^a} \right)^{C_b}$$

(140)

where $K_{\text{sat}}^a$ (m s$^{-1}$) is the aquifer drainage parameter, $S_{\text{crit}}^a$ (m) is a scaling parameter, and $c_b$ (-) is the baseflow exponent.
4. Summary

This note describes the conservation equations and flux parameterizations used in the Structure for Unifying Multiple Modeling Alternatives (SUMMA). SUMMA is based on the community understanding of how the dominant fluxes of energy and water affect the time evolution of thermodynamic and hydrologic states. This provides scope to define a common set of conservation equations, that are shared across multiple models, and make a set of general spatial approximations that preserve the flexibility to experiment with different modeling alternatives [Clark et al., 2015a; Clark et al., 2015b]. The different process representations described here are integrated into the structural model core, which enables users to decompose the modeling problem into the individual decisions made as part of model development and evaluate different model development decisions in a systematic and controlled way.

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Appendix A  Constitutive functions for melt-freeze

The LHS of the thermodynamic state equations (2) and (3) include two state variables, the temperature $T$ and the volumetric ice content $\theta_{ice}$, and, in order to close the equations, additional equations are needed to relate $T$ to $\theta_{ice}$.

A.1 Relationships between $\theta_{liq}$ and $T$ for vegetation and snow

The relationship between $\theta_{liq}$ and $T$ for the vegetation canopy and the snowpack uses the empirical function

$$\theta_{liq} (T, \Theta_m) = \frac{\Theta_m}{1 + \left(\sigma \left(T_{frz} - T\right)\right)^2} \tag{A1}$$

where $\Theta_m$ (-) is total equivalent liquid water content, i.e., $\Theta_m = \theta_{liq} + \rho_{ice} \theta_{ice} \rho_{liq}$, $\sigma$ (K$^{-1}$) is a scaling parameter, and $T_{frz}$ (K) is the freezing point of pure water. Equation (A1) is a simplified version of the freezing curve used by Jordan [1991], and defines a smoothed step function reaching a maximum value at $T_{frz}$. The parameter $\sigma$ hence controls the degree of smoothing, with $\sigma = 50$ being visually indistinguishable from a step function over a typical diurnal temperature range.

A.2 Relationships between $\theta_{liq}$ and $T$ for soil

In this work we combine the generalized Claperyon equation with the water retention curve in order to separate the total water content $\Theta_m$ into the volumetric fractions of liquid water $\theta_{liq}$ and ice $\theta_{ice}$.

The generalized Claperyon equation can be expressed for the state of water as

$$\frac{dP_w}{dT} = \frac{\rho_{liq} L_{fus}}{T} \quad T < T_* \tag{A2}$$

where $P_w$ (Pa) is the water pressure, $T_*$ is the freezing temperature (note that $T_* < T_{frz}$ ) and $L_{fus}$ (J kg$^{-1}$) is the latent heat of fusion. The liquid water matric potential from the Claperyon equation $\psi_{cw}$ can then be computed during sub-freezing conditions as (noting that $P_w = \rho_{liq} \cdot g \cdot \psi$)

$$\psi_{cw} (T) = \frac{L_{fus} (T - T_f)}{g \cdot T_f} \quad T < T_* \tag{A3}$$
where \( g \) (m s\(^{-2}\)) is the acceleration due to gravity (see Dall’Amico et al., [2011] and Painter and Karra [2014] for derivations). Here we follow the approach adopted by Zhao et al. [1997] and only use the Claperyon equation to separate the phases within soil (i.e., \( \psi_{cc} \neq \psi \)).

The freezing temperature \( T_\ast \) can then be obtained by setting \( \psi_{cc} = \psi_0 \) in equation (A3) and solving for \( T \) [Dall'Amico et al., 2011]

\[
T_\ast(\psi_0) = T_f + \frac{g T_f}{L_{fus}} \psi_0
\]

(A4)

The volumetric liquid water content can then be computed as

\[
\theta_{\text{liq}}(\psi_0, T) = \begin{cases} 
S_s \left( \frac{L_{fus} T - T_f}{g T_f} \right) & T < T_\ast \\
S_s(\psi_0) & T \geq T_\ast 
\end{cases}
\]

(A5)

where \( S_s(\cdot) \) is the desired water retention curve, e.g., the van Genuchten [1980] function, defined as

\[
S_s(\psi_{cc}) = \frac{\theta_{\text{sat}} - \theta_{\text{res}}}{1 + (\alpha_{rg} \psi_{cc})^{n_{rg}}} + \theta_{\text{res}}
\]

(A6)

with \( \alpha_{rg} \) (m\(^{-1}\)) the capillary length scale, and the parameters \( m_{rg} \) (-) and \( n_{rg} \) (-) are related to the pore size distribution (\( m_{rg} = 1 - 1/n_{rg} \)).

The volumetric ice content can then be defined based on volume constraints, with

\[
\theta_{\text{air}}(\psi_0) = \theta_{\text{sat}} - S_s(\psi_0)
\]

(A7)

\[
\theta_{\text{ice}}(\psi_0, T) = \theta_{\text{sat}} - \theta_{\text{air}} - \theta_{\text{liq}}
\]

(A8)
References


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