Digital Control of Meteorological Systems

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PREFACE

This report documents the calculations that are necessary to enable applications of digital control methods to the sampled-data systems used for meteorology.

ABSTRACT

The state-variable matrix approach of modern control theory is applied to a multi-input, multi-output meteorological network. The problem of providing suitable controllers to stabilize the outputs in the presence of deteriorating input parameters is discussed. Numerical simulation of an illustrative example was performed on a digital computer.

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INTRODUCTION

Even after the configuration of a meteorological observational network is established, the problems can be encountered in the subsystems:

- Abrupt failure, with adverse effects on transferral of data.
- Drift-type degradation, in which the subsystems continue to function but performance becomes so degraded that output characteristics drift beyond specifications and reliable information is difficult to gather from derived data.

Because situations of the second type are more commonly encountered, the discussion in this report will be devoted to the problem of providing suitable compensators to stabilize the derived outputs.
ANALYTICAL TREATMENT

The central objective here is to review the evaluation methods involving z-transform techniques. The analysis will be similar to the structural approach for multivariable process control in reactor systems.

STATE MODEL APPROACH

A meteorological observational network (Fig. 1) is a sampled-data system with multiple inputs and outputs. Thus, we have to consider a multivariable system with M outputs and N inputs, where \( x(t) \), \( y(t) \), \( z(t) \), and \( e(t) \) are the input, output, desired output, and error vectors, respectively. \( x(t) \) is an \((N \times 1)\) vector, and \( y(t) \), \( z(t) \), and \( e(t) \) are \((M \times 1)\) vectors.

It is evident that the general problem of minimizing the objectionable effects of \( e \) on overall system performance is nonlinear. To provide a workable scheme, however, it will be assumed that the structure of the meteorological system permits the output characteristics to be described by the state model (Gupta and Hasdorff, 1970) whose general form is

\[
\frac{d}{dt} \{\dot{y}\} = [A]\{\dot{x}\} + [B]\{\dot{y}\} \tag{1}
\]

where \( \{\dot{y}\} \) is defined as the state vector and \([A]\) and \([B]\) are coefficient matrices. \( \{\dot{x}\} \) is a column vector whose elements are the input variables (perturbations). Each component of the state vector represents the output. Thus, if the values for \( \{\dot{y}(t)\} \) are known, the state of the system is completely defined. The approximate-linear system evolves in time, according to the relation

\[
\{y(t)\} = \int_{t_0}^{t} e^{[B](t-t')}[A]\{x(t')\}dt' + e^{[B](t-t_0)} \{y(t_0)\} \tag{2}
\]
Here \( \{y(t)\} \) is the state of the system at some initial time \( t_0 \). The transition-matrix \( \exp\{[B](t-t_0)\} \) is defined by the infinite series

\[
\exp\{[B](t-t_0)\} = [I] + [B](t-t_0) + [B]^2 \frac{(t-t_0)^2}{2!} + \ldots
\]

\[
+ [B]^n \frac{(t-t_0)^n}{n!} + \ldots
\]

where \([I]\) is the unit-matrix.

Let us assume that there are \( c \) controlled outputs. To facilitate the analysis, we will reduce the input vector \( \bar{x} \) to \( k \) known inputs, and \( m \) manipulable inputs. In partitioned form, we can write the state model in two parts as

\[
\begin{align*}
\{y_c(t)\} &= e^{[B](t-t_0)}\{y_c(t_0)\} + \int_{t_0}^{t} e^{[B](t-t')} [A] \begin{bmatrix} x_k \\ x_m \end{bmatrix} dt'
\end{align*}
\]

**DIGITAL CONTROL SCHEME**

We propose to stabilize the output characteristics in the presence of monitored input perturbations introduced by \( x_k \). Therefore, we test digital control methods for decoupling the built-in interactions of the multi-element set by providing appropriate feed-forward and feedback controllers.

In meteorological systems, inputs and outputs are time sequences instead of time functions. For time-instant inputs, it is reasonable to assume that output disturbances occur at discrete instants of time \( t = nT \). Between the sampling intervals \( nT \) and \((n+1)T\), we will assume that the output is constant, and also,

\[
\begin{align*}
x_m(t) &= x_m(nT) \quad \text{for} \quad nT \leq t \leq (n+1)T
\end{align*}
\]
for $t_o = nT$ and $t = (n+1)T$, we can write Eq. (4) as

$$\{y_c(nT+T)\} = e^{[B]T}\{y_c(nT)\} + \int_{nT}^{(n+1)T} dt' e^{[B](nT-T+t')}$$

$$\times [A^{ck}]\{x_k(t')\} + \int_{nT}^{(n+1)T} dt' e^{[B](nT-T+t')} [A^{cm}]\{x_m(t')\}$$

(6)

where $A^{ck}$ and $A^{cm}$ are $c \times k$ and $c \times m$ submatrices associated with $x_k(t')$ and $x_m(t')$, respectively.

The actual input to a meteorological system, the noise interference, and the actual system outputs are usually of stochastic nature and can, in general, be described only statistically. Therefore, a precise cancellation of objectionable input perturbations is not possible because the regulating signal that will compensate for $x_k(t)$ during the interval $nT$ to $(n+1)T$ is not clearly defined at $t = nT$. Consequently, the problem will be treated as an extrapolated deterministic case.

An extrapolated continuous time function which coincides with $x_k(t)$ at sampling instants (Freeman, 1965) will be taken to be of the form

$$\{\tilde{x}_k\} = \sum_{\ell=0}^{L} [w_\ell(t-nT)]\{x_k(nT-\ell T)\}$$

(7)

where $w_\ell$ is the weighting function for $x_k(nT-\ell T)$. Equation (6) can now be rewritten, using Eq. (7), as

$$\{y_c(nT+T)\} = [b^{cc}]\{y_c(nT)\} + \sum_{\ell=0}^{L} [\tilde{A}^{ck}_\ell]\{x_k(nT-\ell T)\}$$

$$+ \{\tilde{x}_k(nT)\} + [\tilde{A}^{cm}]\{x_m(nT)\}$$

(8)
where

\[
\begin{align*}
[b^{cc}] &= \exp([B]T) \\
[\tilde{A}_{ck}] &= \int_{nT}^{(n+1)T} dt' \, e^{[B](nT+t'-T')} \, [A^{ck}] \, w_{k}(t'-nT) \\
[\tilde{A}_{cm}] &= \int_{nT}^{(n+1)T} dt' \, e^{[B](nT+t'-T')} \, [A^{cm}] \\
[\tilde{y}_k] &= \int_{nT}^{(n+1)T} dt' \, e^{[B](nT+t'-T')} \, [A^{ck}] \{x_k(t') - \tilde{x}_k(t')\}
\end{align*}
\]

The $z$-transform of the output can now be written as

\[
\begin{align*}
z\{y_c(z)\} &= [b^{cc}] \{y_c(z)\} + \left[I_c \, A^{ck}(z) \, A^{cm}(z)\right] \begin{bmatrix} x_k \\ x_m \end{bmatrix} \\
&= z\{x(t)\} = X(z)
\end{align*}
\]

where

\[
\beta[x(t)] = X(z)
\]

and

\[
A^{ck}(z) = \sum_{\ell=v}^{L} A^{ck}_{\ell} z^{-\ell}
\]

In Eq. (10), $[b^{cc}]$ denotes the "feedback matrix"; the off-diagonal elements of this matrix represent feedback intercoupling of input and output variables. We are interested in the design of feedback decoupling controls to suppress the adverse effects of disturbances upon the
output of the controlled system. Let us define the "feedback inter-
coupling matrix" as

$$[b_{fb}^{cc}] = [b^{cc}] - [b_{d}^{cc}]$$  \(12\)

where \([b_{d}^{cc}]\) is the diagonal matrix obtained from the diagonal elements of \([b^{cc}]\). Making use of Eq. (12), Eq. (10) is reduced to

$$\{Y_{c}\} = [G_{d}^{cc}](z^A c k A^{cm}) \{X_{k}\} + [b_{fb}^{cc}]\{Y_{c}\}$$  \(13\)

with the subsystem transfer matrix defined by

$$[G_{d}^{cc}] = (z[I_{c×c} - [b_{d}^{cc}])^{-1}$$  \(14\)

A graphical representation of the equations describing this system is presented in the signal flow graph of Fig. 2.

Clearly, the above formulation will achieve the control functions only at discrete instants. If the selected sampling intervals are reasonably small, the regulating function can be achieved for all time. We will also assume that the sensor time delays are small in comparison with the time constants of the controlled network. In real systems, these assumptions may or may not be entirely valid. After we gain some insight into the feasibility criteria under these simplifying assumptions we may revise them for future work.

Introducing \(A^{cm}_{d}\) as the diagonal matrix composed of diagonal elements of \(A\), Eq. (13) can be recast as

$$\{Y_{c}(z)\} = [\hat{G}^{cc}(z)](\hat{A}^{ck}(z) \hat{A}^{cm}(z)) \{X_{k}\}$$

$$+ [\hat{b}^{cc}(z)]\{Y_{c}(z)\}$$  \(15\)
in which
\[
\begin{align*}
\left[ \hat{G}^{cc}(z) \right] &= \left[ G^{cc} \right] \left[ A_{cm}^{d} \right] \\
\left[ \hat{A}^{cm}(z) \right] &= \left[ A_{cm}^{d} \right]^{-1} \left[ A_{cm}^{*} \right] \\
\left[ \hat{A}^{ck}(z) \right] &= \left[ A_{cm}^{d} \right]^{-1} \left[ A_{cm}^{*ck} \right] \\
\left[ \hat{b}^{cc}(z) \right] &= \left[ A_{cm}^{d} \right]^{-1} \left[ b_{fb}^{cc} \right]
\end{align*}
\] (16)

Analytical design of the required regulating function to suppress sub-system disturbances may be carried out using the invariance concept (Powers and Ward, 1970). In a signal flow graph in which two nodes are connected by a forward branch transfer function \( G \), the requirement for the invariance of the second node in the presence of disturbances in the first is a parallel control branch with the transfer function \(-G\). We then obtain, for the regulating function,
\[
\begin{align*}
\{ X_{m}(z) \} &= \left[ F_{mk}(z) \right] \{ X_{k}(z) \} + \left[ F_{mm}(z) \right] \{ X_{m}(z) \} + \left[ F_{mc}(z) \right] \{ Y_{c}(z) \} \\
\end{align*}
\] (17)

where
\[
\begin{align*}
\left[ F_{mk}(z) \right] &= - \left[ \hat{A}^{ck}(z) \right] \\
\left[ F_{mm}(z) \right] &= - \left[ \hat{A}_{cm} \right] \\
\left[ F_{mc}(z) \right] &= - \left[ b^{cc}(z) \right]
\end{align*}
\] (18)

The physical significance of control functions \( F \) is as follows:
- \[ F_{mk} \] provides suitable adjustment of manipulable inputs to counteract the disturbances in known inputs.
- \[ F_{mm} \] suppresses forward intercouplings through manipulable inputs.
- \[ F_{mc} \] is essentially a feedback decoupler; perturbing influences on a particular output variable due to unknown inputs are not transferred to other output variables.

Figure 3 shows the signal flow graph incorporating the control functions.
To clarify the above general discussions, a specific example will be treated. Meteorological characteristics are generally measured or deduced from sensors either attached to or consisting of a moving observational platform (e.g., aircraft, balloon, satellite, or rocket). Analysis of the data provided by the observational systems yields observed point quantities such as temperature, specific humidity, and normal wind velocity at specific points along the path of the sensing system. These specific quantities are then used to obtain derived point quantities and possibly field data. Let us consider an inter-aircraft calibration experiment as an illustrative case. Let $X_1$, $X_2$, and $X_3$ be the known inputs (measured quantities). We will assume that the controlled outputs $D_1$ and $D_2$ are the derived quantities from measurements of $X_1$, $X_2$, and $X_3$. We will also assume that random disturbances in $X_1$, $X_2$, and $X_3$ can be monitored.

The state model of our multi-element system can be written in a matrix format

$$\frac{dx}{dt} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \quad \ldots \quad (19)$$

where the variables will now be taken to be the perturbations. The $A$'s are input variable coefficients (see Eq. 1). In this example, $k = 3$, $x_k = (X_1, X_2, X_3)$; $m = 2$, $x_m = (X_4, X_5)$, the manipulated corrections to $X_1$, $X_2$; and $y_c = (D_1, D_2)$. The $A_{mn}$'s represent the effect of nth
input on the mth output. To evaluate the control functions, $F$, the z-transformed version of Eq. (19) is required.

Making the assumption that $x_k(t)$ is a piecewise constant function changing in magnitude at discrete points in time $(T, 2T, \ldots nT, \ldots)$, we obtain from Eq. (9)

\[
\begin{align*}
\begin{bmatrix}
\tilde{A}^{ck} \\
\tilde{A}^{cm}
\end{bmatrix} &= \int_{nT}^{(n+1)T} dt' \exp\{B(nT+T-t')\} \begin{bmatrix}
A^{ck} \\
A^{cm}
\end{bmatrix} \\
\end{align*}
\]

The formula describing the input-output characteristics of the observational network is of the form (see Eq. 8)

\[
\begin{align*}
\begin{bmatrix}
D_1(n+1) \\
D_2(n+1)
\end{bmatrix} &= \begin{bmatrix}
b^{cc} \\
\end{bmatrix} \begin{bmatrix}
D_1(n) \\
D_2(n)
\end{bmatrix} + \begin{bmatrix}
\tilde{A}^{ck} \\
\tilde{A}^{cm}
\end{bmatrix} \begin{bmatrix}
x_3 \\
x_4 \\
x_5
\end{bmatrix} \\
\end{align*}
\]

where for convenience $y_c[(n+1)T]$ is written simply as $y_c(n+1)$.

The z-domain state model obtained by taking the z-transform of Eq. (21) is

\[
\begin{align*}
\begin{bmatrix}
Y_D(z) \\
Y_D(z)
\end{bmatrix} &= [G_{2x2}] \begin{bmatrix}
A^{ck}(z) \\
A^{cm}(z)
\end{bmatrix} \begin{bmatrix}
x_3 \\
x_4 \\
x_5
\end{bmatrix} \\
&+ \begin{bmatrix}
b^{cc} \\
\end{bmatrix} \begin{bmatrix}
Y_D(z) \\
Y_D(z)
\end{bmatrix} \\
&\ldots
\end{align*}
\]
Eq. (22) can now be normalized to the final structural form of Eq. (15). In this manner, the control-regulating functions $F$ can be derived to uncouple the derived quantities $D_1$ and $D_2$ and to provide feed-forward compensation for disturbances entering $X_1$, $X_2$, and $X_3$.

The interconnections between the multiple inputs and outputs for the present example are shown in Fig. 4. The "controller" (a digital computer) activates corrective signals $X_m$ to stabilize the perturbations in the derived outputs. The $z$-transformed system graph of the multi-element systems evaluation model (MESEM) incorporating digital controllers is indicated in Fig. 5.

A program to evaluate the effect of feed-forward and feedback control for the foregoing example has been written in the FORTRAN language and used in the GE Mark II time-sharing computer. The computing routine for the state-variable method is summarized in the flow chart of Fig. 6.

The computer simulation was carried out using

$$\begin{bmatrix} A^{ck} & A^{cm} \\ \end{bmatrix} = \begin{bmatrix} .2 & .2 & .2 & .2 \\ .1 & .1 & .1 & .4 & .3 \end{bmatrix}$$

The choice of the matrix $[B]$ should be such that the Schur-Cohn stability criterion (Tou, 1959) is satisfied. Numerical calculations are made with $[B]$ in the form

$$\begin{bmatrix} 0 & 1 \\ -ab & -(a+b) \end{bmatrix}$$

so that $F(Z) = Z^2 - (a+b)Z + ab$ satisfies the following conditions:

$$|F(0)| < 1, \quad F(1) > 0, \quad F(-1) > 0.$$

When all these conditions are satisfied, control stability is ensured. For the present set of calculations the values chosen were $a = 0.5$ ($\sqrt{25\%}$) and $b = 0.866$ ($\sqrt{75\%}$). The values in parentheses signify that the intensity effect of $D_2$ on $D_1$ is assigned to be about 25% and the effect of $D_2$ on itself is taken to be about 75%.
Computer runs were made for various values of $T/T_d$, where $T$ is the sampling period and $T_d$ is the predominant time constant of the system. Figure 7 shows the output sequence $D$ plotted in response to piecewise constant excitation vectors $X_k$, namely, $e_{X_k} (nT) = nT/2 \ [nT<t<(n+1)T]$. The effectiveness of the controllers is considered in terms of the overshoot and the number of periods required for $D$ to settle to its final value. For $T > 0.35$, it requires more than two periods for $D$ to settle to a final constant level and the percentage overshoot is relatively higher. For higher $T$, the inter-rippling between successive instants is also more pronounced, though $D$ tends to get smaller for higher $nT$.

The effect of setting $F_{mc} = 0$ ($F_{mk}, F_{mm} \neq 0$) is depicted in Fig. 8 for $T = 0.2$. The final value of $D$ is about three times higher than that for the case with $FMC \neq 0$. Thus the feedback controller $F_{mc}$ is indeed effective in reducing the magnitude of $D$. 
CONCLUSIONS

On the basis of preliminary studies we conclude that the state variable matrix approach appears to be a useful method for treating interacting meteorological subsystems. A foreseeable application of the proposed method could be in the evaluation of anemometer sensor/transducer/recording systems.¹

However, we should point out a few limitations of the method. The physical significance of the $A_{ij}$'s has been assumed to represent the fractional contribution of the $j$th output to the net deterioration in the $i$th output. In some actual systems, it may not be possible to apportion the fractional contributions in a deterministic way. The choice of $B$, on the other hand, is limited by the stability requirements of control theory. Sometimes this might demand more information for the specification of the state model than is available in the actual experiment.

As the simulation of large, complex systems is a lengthy process, the state variable methods have potential uses for real-time applications only for relatively small systems, mainly because $z$-transform methods require all operations to be accomplished in the complex-variable domain.

To broaden the design of physically acceptable controllers in a multi-element array of meteorological networks, consideration should be given to general types of subsystem artifacts which can be described only statistically. Research on statistical design using a systems approach and the mean square error criterion (Gupta and Hasdorff, 1970) to optimize the transfer function is already in progress.

¹C. Duchon, personal communication, 1971.
Fig. 1 Block diagram of a meteorological measurement system.

Fig. 2 Signal flow graph for Eqs. (10) and (13). $Z_c$ denotes a summing node.
Fig. 3 Signal flow graph with the regulating functions, F.

Fig. 4 Schematic presentation of a meteorological system using a stabilizing controller
Fig. 5  z-transformed MESEM incorporating digital controllers.
Specify sampling periods, nature of input perturbations coefficient matrices; choose $[B]$ to meet stability requirements of control theory

Estimate the transition matrix $\exp{[BT]}$

Set up Z-transformed structure and calculate the transfer function $G$

Calculate the control functions $F_{MK}, F_{MM}, F_{MC}$.
Find $X_M(z)$ using the $F$'s.

Invert to find the manipulated inputs in the time domain
Find $y_c((n+1)T)$ using $X_M(nT)$

Compute output measures of effectiveness of controllers

Fig. 6 Flow chart for the simulation of MESEM on a digital computer.
Fig. 7 Output sequence in response to piecewise constant excitation.
Fig. 8 Effect of feedback controller in reducing output perturbation.
REFERENCES


